Macroeconomic Risks and Asset Pricing: Evidence from a Dynamic Stochastic General Equilibrium Model

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Abstract

We study the relation between macroeconomic fundamentals and aggregate stock market returns through the lens of a state of the art dynamic stochastic general equilibrium (DSGE) model considered in Christiano, Trabandt and Walentin (2011). We provide a full-information Bayesian estimation of the model using macro variables and extract three fundamental shocks to the economy through the model: neutral technology shock, investment-specific technological shock, and monetary policy shock. While it has been shown that the DSGE model matches a wide range of macroeconomic variables well, we are the first to show that the three shocks have significant and robust predictive power of aggregate stock market returns. Compared to other predictors of stock returns, such as cay of Lettau and Ludvigson (2001) and output gap of Cooper and Priestley (2009), the three shocks are obtained from a structural model, closer to economic fundamentals and represent more exogenous shocks to the economy. Our results show that DSGE models, which have been successful in modeling macroeconomic dynamics, have great potential in capturing asset price dynamics as well.

1 Introduction

One of the key issues in asset pricing is to understand the economic fundamentals that drive the fluctuations of asset prices. Modern finance theories on asset pricing, however, have mainly focused on the relative pricing of different financial securities. For example, the well-known Black-Scholes-Merton option pricing model considers the relative pricing of option and stock while taking the underlying stock price as given. The celebrated Capital Asset Pricing Model (CAPM) relates individual stock returns to market returns without specifying the economic forces that drive market returns. Modern dynamic term structure models also mainly focus on the relative pricing of bonds across the yield curve. These models tend to assume that the yield curve is driven by some latent state variables without explicitly modeling the economic nature of these variables.

Increasing attention has been paid in the literature to relate asset prices to economic fundamentals as evidenced by the rapid growth of the macro finance literature. For example, the macro term structure literature has been trying to relate term structure dynamics to macro fundamentals. By incorporating the Taylor rule into traditional term structure models, several studies have shown that inflation and output gap can explain a significant portion of the fluctuations of bond yields. The investment based literature has also tried to relate equity returns to firm fundamentals, thus giving economic meaning to empirical based factors (such as HML and SMB) for equity returns. Current attempts to connect macro variables with asset prices, however, are typically based on partial equilibrium analysis. Without a well specified general equilibrium model, it is not clear that the exogenously specified pricing kernels in these "reduced-form" models are consistent with general equilibrium. It is also difficult to identify any causal relations among government policies, macro variables, and asset prices. Given that financial assets are claims on real assets, explicit general equilibrium modeling of the whole economy might help to better understand the economic forces that drive asset prices.

The New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models offer such a framework to understand the link between asset prices and economic fundamentals. DSGE models have become a dominant modeling framework in macroeconomics and have been widely used by both academics and central bankers around the world for policy analysis, (see, e.g., Clarida, Galí and Gertler (2000) and Gali and Gertler (2007)). Under the sticky price equilibrium of these models, monetary policy is not neutral and has important impacts on the real activities of the economy and direct implications for the prices of financial assets. However, most existing studies on DSGE models in the macroeconomic literature, such as Christiano, Eichenbaum and Evans (2005), Clarida, Galí and Gertler (2000) , and Smets and Wouters (2007), have mainly focused on the real sector and ignored the financial sector. The recent global financial crisis has highlighted the importance of the financial sector for the stability of the global economy. A good DSGE model should be able to capture the financial sector and consequently asset prices well. Therefore, financial prices provide an alternative perspective to examine potential shortcomings of DSGE models: If they make counter factual predictions on financial prices, then one should be careful in using them for policy analysis. Since financial prices are forward looking and contain market expectations for future economic activities, we can also better identify model parameters and policy shocks by incorporating financial prices in the estimation of DSGE models.

In this paper, we study the link between macroeconomic fundamentals and aggregate stock returns through the lens of New Keynesian DSGE models. In particular, we study whether fundamental economic shocks considered in these models have any explanatory power of aggregate stock market returns. Given that stocks represent claims on real productive assets, important drivers of economic growth and business cycle should also affect the fluctuations of stock returns. For example, total factor productivity represents the overall efficiency of capital and labor in producing goods and services, while investment-specific technological shock represents the efficiency of machines and equipments. Non-neutral monetary policy also has significant impact on real economic activities. Therefore, at least theoretically there should be close connections between these macroeconomic factors and stock returns.

Our analysis is based on a state of the art DSGE model considered in Christiano, Trabandt and Walentin (2011) (CTW), which includes all the major ingredients of DSGE models. CTW have shown that this model matches a wide range of macroeconomic variables very well. In this paper, we provide one of the first studies that examines the ability of this DSGE model in explaining aggregate stock market returns. Our paper makes several important contributions to the macro literature on DSGE models as well as the finance literature on asset pricing.

First, we develop full-information Bayesian Markov Chain Monte Carlo (MCMC) methods for estimating DSGE models using macroeconomic variables. Whereas the Bayesian moment matching methods of CTW essentially match the unconditional moments of the macro variables, our full-information Bayesian MCMC methods fully exploit the conditional information contained in the likelihood function of the macro data. As a result, our methods provide more efficient estimation of model parameters. More important, our MCMC methods make it possible to back out the latent shocks to the economy in DSGE models. In contrast, the Bayesian moment matching methods cannot back out the latent shocks because they can only match the long-run average features of the data.

Second, we estimate the DSGE model of CTW using our full-information Bayesian MCMC methods based on macroeconomic variables only. We obtain reasonable estimates of model parameters and confirm the findings of CTW that the DSGE model can match a wide range of macro variables well. In addition, we back out the three fundamental shocks to the economy in the DSGE model, namely the neutral technology (NT) shock, the investment-specific technological (INV) shock, and the monetary policy (MP) shock.

Finally, we examine the predictive power of the three extracted shocks of aggregate stock market returns. We regress the CRSP value-weighted index return on NT, INV, and MP shocks. The whole sample period is from the first quarter of 1966 to the third quarter of 2010. We use the three shocks to forecast future one-month, one-quarter, and one-year return of the CRSP index. In general, we find all three shocks have strong predictive power of future stock returns, although the INV shock becomes more significant at longer forecasting horizon. Welch and Goyal (2007) have shown that predictability of stock returns tend to be sensitive to sample period used. To test the robustness of our results, we change the starting date of the sample period to the first quarter of 1970 and 1975 and obtain very similar results. We also compare the predictive power of the three shocks with that of other predictive variables that have been studied in the literature, such as the *cay* factor of Lettau and Ludvigson (2001) and output gap (*gap*) of Cooper and Priestley (2009). We find that our three shock variables have much stronger and more robust predictive power than *cay* and *gap*.

Our result is a testament of the power of the DSGE approach. Given that we estimate the DSGE model using only macro data, it is amazing that the three shocks extracted from the model have such strong predictive power of stock returns. The three shocks have important advantages over other predictive variables considered in the literature. First, they are derived from a structural economic model and therefore have clear economic meaning. Second, they represent more fundamental forces in the economy. In contrast, *cay* and *gap* are derived rather than fundamental variables. Third, the three shocks represent more exogenous forces to the economy. Finally, the most important advantage of our approach is that it shows that the DSGE approach captures important elements of the economy such that the shocks extracted from the model can predict asset returns even out of sample. Therefore, it highlights the possibility of integrating macroeconomics and asset pricing under an unified modeling framework.

The rest of the paper is organized as follows. Section II introduces the DSGE model. Section III discusses the full-information Bayesian estimation methods. Section IV discuss the data and empirical results, and Section V concludes.

2 The Model

The DSGE model that we estimate is taken from CTW. The modeled economy contains a perfectly competitive final goods market, a monopolistic competitive intermediate goods market, households who derive utility from final goods consumption and disutility from supplying labor to production. There are Calvo (1983) type of nominal price rigidities and wage rigidities in the intermediate goods market. Government consumes a fixed fraction of GDP very period and the monetary authority set the nominal interest rate according to a Taylor rule. There are three exogenous shocks in the economy: total factor productivity shocks, investment-specific technological shocks, and monetary policy shocks. CTW show that the model matches vey well an important set of macroeconomic variables including: changes in relative prices of investment, real per hour GDP growth rate, unemployment rate, capacity utilization, average weekly hours, consumption-to-GDP ratio, investment-to-GDP ratio, job vacancies, job separation rate, job finding rate, weekly hours per labor force, Federal Funds Rates. Next, we present the model in details.

2.1 Production sector

There are two industries in the production sector, final goods industry and intermediate goods industry. The production of the final consumption goods uses a continuum of intermediate goods, indexed by $i \in [0, 1]$ via the Dixit-Stiglitz aggregator

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} \, di \right]^{\lambda_f} , \quad \lambda > 1 , \tag{1}$$

where Y_t is the output of final goods, $Y_{i,t}$ is the amount of intermediate goods i used in the final good production, which in equilibrium equals the output of intermediate goods i, and λ_f measures the substitutability among different intermediate goods. The larger λ is, the more substitutable the intermediate goods are. Since the final goods industry is perfectly competitive, profit maximization leads to the demand function for intermediate goods i:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{\lambda_f - 1}} , \tag{2}$$

where P_t is the nominal price of the final consumption goods and P_{it} is the nominal price of intermediate goods i. It can be shown that goods prices satisfy the following relation:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\lambda_f - 1}} di \right)^{-(\lambda_f - 1)} . \tag{3}$$

The production of intermediate goods i employs both capital and labor via the following homogenous production technology

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} K_{i,t}^{\alpha} - z_t^{+} \varphi \, , \tag{4}$$

where z_t is the neutral technology shock, $H_{i,t}$ and $K_{i,t}$ are the labor service and capital service, respectively, employed by firm i, α is the capital share of output, and φ is the fixed production cost. Finally, z_t^+ is defined as

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t \, ,$$

where Ψ_t is the investment-specific technology shock, measured as the relative price of consumption goods to investment goods. Assume that the neutral technology shock z_t and Φ_t evolve as follows:

$$\mu_{z,t} \;=\; \mu_z + \rho_z \, \mu_{z,t-1} + e_t^{z} \, , \quad \text{where } \mu_{z,t} = \Delta \log z_t \, , \, \mathbb{E}\left(e_t^{z}\right) = \sigma_z^2 \, , \tag{5}$$

$$\mu_{\psi,t} \;=\; \mu_\psi + \rho_\psi \, \mu_{\psi,t-1} + e_t^{\psi} \, , \quad \text{where } \mu_{\psi,t} = \Delta \log \psi_t \, , \, \mathbb{E}\left(e_t^{\psi}\right) = \sigma_\psi^2 \, . \tag{6}$$

The intermediate goods industry is assumed to have no entry and exit, which is ensured by choosing a fixed cost ψ that brings zero profits to the intermediate goods producers.

Intermediate goods producer i rents capital service K_{it} from households and its net profit at period t is given by

$$P_{it}Y_{it} - r_t^K K_{it} - W_t H_{it}\,.$$

The producer takes the rent of capital service r_t^K and wage rate W_t as given but has market power to set the price of its goods in a Calvo (1983) staggered price setting to maximize its profits. With probability ξ_p, producer i cannot reoptimize its price and has to set its price according to the following rule,

$$P_{i,t} = \pi\, P_{i,t-1}$$

and with probability $1-\xi_p$, producer i sets price $P_{i,t}$ to maximize its profits, i.e.,

$$\max_{\{P_{i,t}\}} \; \mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\xi_p\,\beta\right)^{\tau} \nu_{t+\tau} \left[P_{i,t}Y_{i,t+\tau\,|\,t} - W_{t+\tau}H_{t+\tau\,|\,t}\right] \tag{7}$$

subject to the demand function in equation (2). In the above objective function, $Y_{i,t+\tau\,|\,t}$ and $H_{t+\tau\,|\,t}$ refer to the output and labor hiring, respectively, by producer i at time $t+\tau$ if the last time when price P_i is reoptimized is period t.

2.2 Households

Following CTW, we assume that there is a continuum of differentiated labor types indexed with j and uniformly distributed between zero and one. A typical household has infinite many members covering all the labor types. It is assume that a household's consumption decision is made based on utilitarian basis. That is, every household member consumes the same amount consumption goods even though they might have different status of employment. CTW show that a representative household's life-long utility can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t - b\, C_{t-1}\right) - A_L \int_0^1 \frac{h_{jt}^{1+\phi}}{1+\phi} \right], \tag{8}$$

subject to the budget constraint

$$P_t \left(C_t + \frac{I_t}{\Psi_t} \right) + B_{t+1} + P_t P_{k',t} \Delta_t \leq \sum_{t=0}^{\infty} \int_0^1 W_{jt} h_{jt}\, dj + X_t^K \bar{K}_t + R_{t-1} B_t \tag{9}$$

for $t = 0, 1, \cdots, \infty$. Here, h_{jt} is the number of household members with labor type j who are employed, B_t is the nominal bond holdings purchased by household at $t-1$, $P_{k',t}$ is the market price of one unit capital stock, X_t^K is the net cash payment to the household by renting out capital $\bar{K}_t$, given by

$$X_t^K = P_t \left[u_t\, r_t^K - \frac{a(u_t)}{\Psi_t} \right] .$$

The wage rate of labor type j is determined by a monopoly union who represents all j-type workers and households take the wage rate of each labor type as given.

Households own the economy's physical capital $\bar{K}$. The amount of capital service K_t available for production is given by

$$K_t = u_t\, \bar{K}_t \,,$$

where u_t is the utilization rate of physical capital and utilization incurs a maintenance cost

$$a(u) = b\, \sigma_a\, u^2/2 + b(1-\sigma_a)\, u + b\left(\sigma_a/2 - 1\right) . \tag{10}$$

where b and σ_a are constants and chosen such that steady state utilization rate is one and at steady state $a(u = 1) = 0$. Note that the maintenance cost $a(u)$ is measured in terms of capital goods, whose relative price to consumption goods is $1/\Phi_t$. A representative household accumulates capital stock according to the following rule:

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + F(I_t, I_{t-1}) + \Delta_t\,,$$

where Δ_t is the capital stock purchased by the representative household and equals zero in equilibrium because all households are identical. Here, $F(I_t, I_{t-1})$ is the investment adjustment cost, defined as

$$F(I_t, I_{t-1}) = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)$$

and

$$S(x_t) = \frac{1}{2}\left\{\exp\left[\sigma_s\left(x_t - \exp(\mu_z^+ + \mu_\psi)\right)\right] + \exp\left[-\sigma_s\left(x_t - \exp(\mu_z^+ + \mu_\psi)\right)\right] - 2\right\}\,,$$

where $x_t = I_t/I_{t-1}$ and $\exp(\mu_z^+ + \mu_\psi)$ is the steady state growth rate of investment. The parameter σ_s is chosen such that at steady state $S(\exp(\mu_z^+ + \mu_\psi)) = 0$ and $S'(\exp(\mu_z^+ + \mu_\psi)) = 0$. Note that investment I_t is measured in terms of capital goods. The consumption goods market clearing is then given by

$$Y_t = C_t + G_t + \tilde{I}_t$$

where G_t is government spending and $\tilde{I}$ is investment measured in consumption goods, which also includes the capital maintenance cost $a(u_t)$, i.e.,

$$\tilde{I} = \frac{I_t + u(a_t)}{\Phi_t}\,.$$

2.3 Labor unions

There are labor contractors who hires all types of labor through labor unions and produce a homogenous labor service H_t, according to the following production function

$$H_t = \left[\int_0^1 h_{jt}^{\frac{1}{\lambda_w}} \, dj \right]^{\lambda_w}, \quad \lambda_w > 1\,, \tag{11}$$

where λ_w measures the elasticity of substitution among different labor types. The intermediate goods producers employ the homogenous labor service for production. Labor contractors are perfectly competitive, whose profit maximization leads to the demand function for labor type i

$$h_{jt} = H_t \left(\frac{W_{jt}}{W_t} \right)^{\frac{-\lambda_w}{\lambda_w - 1}} \tag{12}$$

It is easy to show that wages satisfy the following relation:

$$W_t = \left(\int_0^1 W_{i,t}^{-\frac{1}{\lambda_w - 1}} dj \right)^{-(\lambda_w - 1)}, \tag{13}$$

where $W_{j,t}$ is the wage of labor type j and W_t is the wage of the homogenous labor service.

Assume that labor unions face the same Calvo (1983) type of wage rigidities. Each period, with probability ξ_w, labor union j cannot reoptimize the wage rate of labor type j and has to set the wage rate according to the following rule

$$W_{jt+1} = \pi_t \mu_{z^+}$$

and with probability $1 - \xi_w$, labor union j chooses W_{jt} to maximize households' utility

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^{\tau} \left[\nu_{t+\tau} W_{jt} h_{t+\tau\,|\,t} - A_L \frac{h_{jt+\tau\,|\,t}^{1+\phi}}{1+\phi} \right] \tag{14}$$

subject to the demand curve for labor type j in equation (12). Here, $\nu_{t+\tau}$ is the marginal utility of one $h_{jt+\tau|t}$ is the supply of type j labor at period $t+\tau$ if the last time that labor union j reoptimizes wage rate W_{jt} is period t.

2.4 Fiscal and Monetary Authorities

Following CTW, fiscal authority in the model simply transfers a fixed fraction g of output as government spending, i.e.,

$$G_t = g\,Y_t\,.$$

Monetary authority sets the level of a short-term nominal interest rate according to the following Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{gdp_t}{gdp}\right)\right] + V_t. \tag{15}$$

where R_t is the short-term interest rate, R is the steady state interest rate, and V_t is the monetary policy shock, which follows the process

$$V_t = \rho_V V_{t-1} + \sigma_V e_t^V, \tag{16}$$

with $e_V \sim \text{IID}\mathcal{N}(0,1)$.

A detailed solution of the model is provided in the appendix of CTW and will not be repeated here.

3 Full-Information BMCMC Estimation

In this section, we develop full-information BMCMC method for estimating the aforementioned DSGE model based on observed macroeconomic variables. We choose seven macroeconomic variables following Smets and Wouters (2007): per capita output growth (dy), per capita consumption growth (dc), per capita investment growth (di), wage growth (dw), logarithm of inflation (π), 3-month T-Bill (r), and average weekly hours per capita(h). The three fundamental exogenous shocks are neutral technology shocks $\{\mu_{z,t}\}$, investment-specific technology shocks $\{\mu_{\psi,t}\}$ and monetary policy shocks $\{V_t\}$, defined in equations (5), (6), and (16). Given the initial states, the time-series of the aforementioned three exogenous shocks completely determine the outcome of the economy.

3.1 Solution of the System

Our goal is to solve and estimate the economic system described in Section 2 using the actual economic outcomes observed in history. The model is solved in Dynare [1] to the second order approximation. Let X_t denote the state variables of the model and classify the variables in X_t into three groups:

- X_t^o: observable endogenous state variables
- X_t^u: unobservable endogenous state variables
- X_t^e: exogenous state variables $= \{\mu_{z,t}, \mu_{\psi,t}, V_t\}$

There are three exogenous shocks $U_t = \{e_t^z, e_t^\psi, e_t^V\}$. The variables evolves according the following rules obtained from solving the model

$$\begin{aligned}
X_t^o &= \Gamma^o\left(X_{t-1}, U_t^e, \mathbf{\Theta}\right) = \Gamma^o\left(X_{t-1}^o, X_{t-1}^u, X_{t-1}^e, U_t^e, \mathbf{\Theta}\right) \\
X_t^u &= \Gamma^u\left(X_{t-1}^o, X_{t-1}^u, X_{t-1}^e, U_t, \mathbf{\Theta}\right) \\
X_t^e &= \Gamma^e\left(X_{t-1}^o, X_{t-1}^u, X_{t-1}^e, U_t, \mathbf{\Theta}\right)
\end{aligned}$$

[1] Please find detailed information on Dynare at www.dynare.org.

where Θ is the vector of model parameters

$$\Theta = [\beta, \phi, b, \alpha, \delta, \eta_g, \xi_p, \xi_w, K, \lambda_f, \lambda_w, \sigma_a, \sigma_s, \pi_{ss}, \rho_k, \rho_\pi, \rho_y, m_z, \mu_\psi, \sigma_z, \sigma_\psi, \sigma_v, \rho_z, \rho_\psi, \rho_v]$$

and Γ^e is determined by the following relation:

$$U_t = \begin{bmatrix} e_t^z \\ e_t^\psi \\ e_t^V \end{bmatrix} = \begin{bmatrix} \left[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z \mu_{z,t-1}\right]/\sigma_z \\ \left[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi \mu_{\psi,t-1}\right]/\sigma_\psi \\ \left[V_t - \rho_v V_{t-1}\right]/\sigma_v \end{bmatrix}.$$

To calculate X_t, we input the observed values of X_{t-1}^o (denoted as $\tilde{X}_{t-1}^o$) and the model generated values of X_t^u, given the exogenous X_t^e, into the above Γ functions. Therefore, we can calculate X_t^u from the initial values X_0, the time series of $\{\tilde{X}_s^o\}_{s=1}^t$, and the exogenous process $\{U_s\}_{s=1}^t$ as

$$X_t^u = \Gamma^{u,(t)}\left(X_0, \{\tilde{X}_s^o\}_{s=1}^t, \{U_s\}_{s=1}^t, \mathbf{\Theta}\right),$$

using Γ^u function iteratively for t times. Consequently, the model generated values for observable endogenous variables can be written as

$$X_t^o = \Gamma^o\left(\tilde{X}_{t-1}^o, \Gamma^{u,(t-1)}\left(X_0, \{\tilde{X}_s^o\}_{s=1}^{t-1}, \{U_s\}_{s=1}^{t-1}, \mathbf{\Theta}\right), X_{t-1}^e, U_t^e, \mathbf{\Theta}\right).$$

Let Υ_t denote the model solution of the observable variables that we would like to match with the actual observation, which may share some common variables with X_t. Our goal is to choose model parameters Θ and latent variables $\{U_t\}_{t=1}^T$ such that Υ_t is as close to Υ_t^{obs} as possible. Assume that

$$\Upsilon_t = \Gamma\left(X_{t-1}, U_t, \mathbf{\Theta}\right),$$

where the endogenous variables X_{t-1} is given by

$$X_{t-1} = \left\{ \tilde{X}^o_{t-1}, \Gamma^{u,(t-1)} \left(X_0, \{\tilde{X}^o_s\}_{s=1}^{t-1}, \{U_s\}_{s=1}^{t-1}, \boldsymbol{\Theta} \right), X^e_{t-1} \right\} .$$

Based on second order approximation in Dynare, Υ_t depends on the state variables last period (X_{t-1}) and the shocks this period (U_t) to the second order, i.e.,

$$\begin{aligned} \Upsilon_t &= \Gamma\left(X_{t-1}, U_t, \boldsymbol{\Theta}\right) \\ &= \Upsilon_{\text{steady}}\left(\Theta\right) + A + B\, X_{t-1} + C\, U_t + D\, \left(X_{t-1} \otimes X_{t-1}\right) + E\, \left(U_t \otimes U_t\right) + F\, \left(X_{t-1} \otimes U_t\right). \end{aligned}$$

where $\Upsilon_{\text{steady}}\left(\Theta\right)$ represents the steady value of Υ_t and $\otimes$ denotes the Kronecker product. We use matrices

$$\Omega(\Theta) \equiv \begin{bmatrix} \Upsilon_{\text{steady}} & A & B & C & D & E & F \end{bmatrix}$$

to summarize the coefficients in the solution for Υ. We denote the coefficient matrices for the solutions of X^u_t as $\Omega_u(\Theta)$, which are given similarly by

$$\Omega_u(\Theta) \equiv \begin{bmatrix} X^u_{\text{steady}} & A_u & B_u & C_u & D_u & E_u & F_u \end{bmatrix} .$$

All the coefficient matrices depend on model parameters Θ.

3.2 Full-information Bayesian estimation

Define the time series of observable variables as Υ^{obs}_t for $t = 1, \cdots, T$, and assume Υ^{obs}_t are observed with independent pricing errors

$$\Upsilon^{obs}_t = \Upsilon_t + \varepsilon_t = \Gamma(X_{t-1}, \mu_t, \Theta) + \varepsilon_t$$

where $\varepsilon_t = [\varepsilon_{1t}, \cdots, \varepsilon_{7t}], \varepsilon_{it} \sim N(0, \sigma_i^2)$ for $i = 1, \cdots, 7$ and Υ_t is the model implied price from the Γ function that is solved numerically using Dynare package. In the Dynare package, we assume

$$[\Upsilon_t \ X_t^{\mu} \ X_t^{o}] = \Gamma(X_{t-1}, \mu_t; \Theta).$$

where the dynamics of μ_t is determined through the following evolution equations

$$\begin{cases} \mu_{z,t} = \mu_z(1-\rho_z) + \rho_z \mu_{z,t-1} + \sigma_z e_t^z \\ \mu_{\psi,t-1} = \mu_\psi(1-\rho_\psi) + \rho_\psi \mu_{\psi,t-1} + \sigma_\psi e_t^\psi \\ V_t = \rho_V V_{t-1} + \sigma_V e_t^V \end{cases},$$

and Θ. Since μ_t $(t = 1, \cdots, T)$ can be uniquely specified by the sequence $(\mu_{z,t}, \mu_{\psi,t}, V_t)$, the main objective of our analysis is to estimate the model parameters, σ_i $(i = 1, \cdots, 7)$ and Θ $(i = 1, \cdots, 7)$, and latent state variables $S_t = [\mu_{z,t}, \mu_{\psi,t}, V_t]$ $(t = 1, \cdots, T)$ using observationΥ_t^{obs} $(t = 1, \cdots, T)$. The biggest challenge of the analysis is that the marginal likelihood based on parameters only has to be obtained by integrating out a very high dimensional function (on the order of $3 \times T$ dimension due to latent state variables), creating extremely heavy computing burdens. However, solving for parameters and latent variables seems most feasible using Bayesian MCMC methods. In contrast to classical statistical theory, which uses the likelihood $L(\Theta) \equiv p(\boldsymbol{\Upsilon}|\Theta)$, Bayesian inference adds to the likelihood function the prior distribution for Θ, called $\pi(\Theta)$. The distribution of $(\boldsymbol{\Upsilon}, \mathbf{S})$ and $\pi(\Theta)$ combine to provide a joint distribution for $(\boldsymbol{\Upsilon}, \mathbf{S}, \Theta)$ from which the posterior distribution of $(\Theta, \mathbf{S})$ given $\boldsymbol{\Upsilon}$ is produced

$$p(\Theta, \mathbf{S}|\boldsymbol{\Upsilon}) = \frac{p(\boldsymbol{\Upsilon}, \mathbf{S}, \Theta)}{\int p(\boldsymbol{\Upsilon}, \mathbf{S}, \Theta) d\mathbf{S} d\Theta} \propto p(\boldsymbol{\Upsilon}, \mathbf{S}, \Theta).$$

In our context, it is

$$
\begin{aligned}
p(\Theta, \mathbf{S}|\boldsymbol{\Upsilon}) &\propto p(\boldsymbol{\Upsilon}|\mathbf{S}, \Theta) \times p(\mathbf{S}|\Theta) \times \pi(\Theta) \\
&= p(\Upsilon_1^{obs}|\mathbf{S}, \Theta) \times p(\Upsilon_2^{obs}|\Upsilon_1^{obs}, \mathbf{S}, \Theta) \times \cdots \times p(\Upsilon_T^{obs}|[\Upsilon_1^{obs}, \cdots \Upsilon_{T-1}^{obs}], \mathbf{S}, \Theta) \\
&\quad \times p(\mathbf{S}|\Theta) \times \pi(\Theta) \\
&\propto \prod_{t=1}^{T}\prod_{i=1}^{7} \frac{1}{\sigma_i} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_z} \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t-1}]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_\psi} \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t-1}]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_V} \exp\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\} \times \pi(\Theta)\,.
\end{aligned}
$$

In general, it is difficult to simulate directly from the above high dimensional posterior distribution. The theory underlying the MCMC algorithms that eases the computational burden is the Clifford-Hammersley Theorem. This theorem states that the joint distribution $p(\Theta, \mathbf{S}|\boldsymbol{\Upsilon})$ can be represented by the complete conditional distributions $p(\Theta|\mathbf{S}, \boldsymbol{\Upsilon})$ and $p(\mathbf{S}|\Theta, \boldsymbol{\Upsilon})$. MCMC algorithm is done iteratively. In each iteration, each parameter is updated based on most recent value of all other parameters and latent variables through sampling from the corresponding complete conditional distribution, and the latent variables at each time t is also updated in the similar fashion. As this is done, the chains converge (theoretically), to the target posterior distribution. Therefore, after a sufficient number of samples, called a burn-in period, the algorithm is then sampling from a converged target posterior distribution. To find parameter estimates, however, requires some additional machinery. Use of calculus methods will only work nicely if the prior distributions are conjugate priors, leading to tractable solutions. However, in our analysis here, parameters and latent variables are involved into likelihood through the Dynare package, which is a "black box" for us, resulting in intractable posterior distributions. We therefore turn to Metropolis Hastings

Algorithm (MH) for updating both Θ and $\mathbf{S}$. The MH algorithm is an adaptive rejection sampling method where candidate draw is proposed and then accepted with probability proportional to the ratio of the likelihood of the proposed draw to the current draw. This means that if the new position has a higher likelihood (defined using the posterior distribution), then the parameter values are updated with probability 1. Alternatively, if they are less likely, the parameter values are updated with probability according to the likelihood ratio. Thus the parameter values will tend to stay near the highest probability regions when being sampled and adequately cover the probability space. The actual steps involved are as follows provide a vector of starting values for the algorithm, $\Theta^{(0)}$, for iteration g,

- Step 1. Specify a candidate distribution, $h(\Theta|\Theta^{(g-1)})$;
- Step 2. Generate a proposed for parameters, $\Theta^* \sim h(\Theta|\Theta^{(g-1)})$;
- Step 3. Compute the acceptance ratio

$$\Upsilon_g = \frac{p(\Theta^*) \times h(\Theta^*|\Theta^{(g-1)})}{p(\Theta^{(g-1)}) \times h(\Theta^{(g-1)}|\Theta^*)}$$

  where $p(.)$ represents a complete conditional distribution;

- Step 4. Generate $u \sim Unif[0, 1]$, then set

$$\Theta^{(g)} = \begin{cases} \Theta^* & if\ \Upsilon_g \geq u \\ \Theta^{(g-1)} & if\ \Upsilon_g < u \end{cases} \quad ;$$

- Step 5. Set g = g + 1 and return to Step 1.

If the candidate distribution is symmetric, the MH algorithm has acceptance ratio equivalent to $\frac{p(\Theta^*)}{p(\Theta^{(g-1)})}$. In implementation, we chose $h(\Theta|\Theta^{(g-1)}) \sim N(\Theta^{(g-1)}, c^2)$ with some constant variance c^2. The MH algorithm is conducted iteratively on each parameter in Θ and on each latent variable at each time point $t = 1, \cdots, T$. In estimation, we draw posterior samples using the above described MCMC, and use the means of the posterior draws as parameter estimates and the standard deviations of the posterior draws as standard errors of the parameter estimates after a bum-in period.

3.3 Positeriors

In this section, we provides a brief description about the priors, posterior distributions, and the updating procedures for parameters and latent variables in our model.

- *Posterior of* $\sigma_i (i = 1, \cdots, 7)$ — Set the prior of σ_i as $\sigma_i^2 \sim IG(a, b)$, where a,b are hyper-parameters. The posterior of σ_i^2 is

$$\sigma_i^2 \sim IG(\frac{T}{2} + a, A)$$

where

$$A = \sum_{t=1}^{T} \frac{1}{2} (\Upsilon_t^{obs}(i) - \Upsilon_t(i))^2 + b\,.$$

- *Posterior of* $\Theta_i (i = 1, \cdots, 25)$ — Set the prior of Θ_i as ${\Theta_i}^2 \sim N(m, M^2)$ where m, M are hyper-parameters. The posterior of Θ_i is

$$\begin{aligned}
p\left(\Theta_i | \Theta_{[-i]}, \mathbf{S}, \boldsymbol{\Upsilon}\right) \quad \propto \quad & \prod_{t=1}^{T} \prod_{i=1}^{7} \frac{1}{\sigma_i} \exp\{-\frac{1}{2\sigma_i^2} [\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \prod_{t=1}^{T} \frac{1}{\sigma_z} \exp\{-\frac{1}{2\sigma_z^2} [\mu_{z,t} - \mu_z(1 - \rho_z) - \rho_z \mu_{z,t-1}]^2\} \\
& \times \prod_{t=1}^{T} \frac{1}{\sigma_\psi} \exp\{-\frac{1}{2\sigma_\psi^2} [\mu_{\psi,t} - \mu_\psi(1 - \rho_\psi) - \rho_\psi \mu_{\psi,t-1}]^2\} \\
& \times \prod_{t=1}^{T} \frac{1}{\sigma_V} \exp\{-\frac{1}{2\sigma_V^2} [V_t - \rho_V V_{t-1}]^2\} \times \pi(\Theta) \times \exp\{-\frac{(\Theta_i - m)^2}{2M^2}\}\,,
\end{aligned}$$

where $\Theta_{[-i]}$ contains the most recent values of other parameters in Θ. In implementation, we simplify the above posterior through abandoning terms that do not depend on Θ_i, and use MH algorithm to update Θ_i.

- *Posterior of* $\{\mu_{z,t}, \mu_{\psi,t}, V_t\}$ $(t = 1, \cdots, T)$ — The posterior distribution of $\mu_{z,t}$ $(for\ 1 \leq t < T)$ is

$$
\begin{aligned}
& p\left(\mu_{z,t}|\Theta, \{\mu_{z,1}, \cdots, \mu_{z,t-1}, \mu_{z,t+1}, \cdots, \mu_{z,T}\}, \{\mu_{\psi,t}\}_{t=1}^{T}, \{V_t\}_{t=1}^{T}, \boldsymbol{\Upsilon}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t+1} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t}]^2\}\,.
\end{aligned}
$$

For $t = T$, the posterior distribution only involves the first two terms in the above equation. Again, MH algorithm is used to update $\mu_{z,t}$. Updating of $\mu_{\psi,t}$ and V_t $(t = 1, \cdots, T)$ are done in the same way. The analogous posterior distribution for $\mu_{\psi,t}$ is,

$$
\begin{aligned}
& p\left(\mu_{\psi,t}|\Theta, \{\mu_{\psi,1}, \cdots, \mu_{\psi,t-1}, \mu_{\psi,t+1}, \cdots, \mu_{\psi,T}\}, \{\mu_{z,t}\}_{t=1}^{T}, \{V_t\}_{t=1}^{T}, \boldsymbol{\Upsilon}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t+1} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t}]^2\}\,.
\end{aligned}
$$

The analogous posterior distribution for V_t is,

$$\begin{aligned}
& p\left(V_t|\Theta, \{V_1, \cdots, V_{t-1}, V_{t+1}, \cdots, V_T\}, \{\mu_{z,t}\}_{t=1}^T, \{\mu_{\psi,t}\}_{t=1}^T, \boldsymbol{\Upsilon}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_V^2}[V_{t+1} - \rho_V V_t]^2\}\,.
\end{aligned}$$

Table 1 presents the estimated posterior means and standard errors of model parameters, close to what CTW find in their estimation. Figure 1 plots the three exogenous shocks.

4 Empirical Results

In this section, we explore the predictive power of the three latent shocks $\{\mu_t^z, \mu_t^\psi, V_t\}$ on stock returns at one-month, one-quarter, and one-year horizon and through comparison with the *cay* factor in Lettau and Ludvigson (2001) and the *gap* factor in Cooper and Priestley (2009). Our latent shocks are estimated using the seven macroeconomic variables for sample period 1966Q1 to 2010Q3 because of the poor quality of macro data before 1966Q1 (Smets and Wouters (2007)). All macroeconomic data is from the DRI data set from WRDS. Market stock returns are proxied by CRSP value-weighted return, taken from Ken French's website.

The *cay* factor is from Ludvigson's website and constructed based on

$$cay_t = c_{n,t} - a^c - \beta_1^c\, a_t - \beta_2^c\, l_t\,,$$

where $c_{n,t}$ is log of nondurable consumption, a_t is log of asset holdings, l_t is log of labor income.

The coefficients in the above equation comes from the following regression

$$c_{n,t} = a^c + \beta_1^c\, a_t + \beta_2^c\, l_t + \sum_{i=-k}^{k} \beta_{1,i}^c\, \Delta a_{t-i} + \sum_{i=-k}^{k} \beta_{2,i}^c\, \Delta l_{t-i}\,.$$

Lettau and Ludvigson (2001) show that the *cay* factor is a good proxy for market expectations for future returns under certain conditions.

The *gap* factor is constructed based on quarterly industry production index (IP), which is also from the DRI data set, according to following regression model

$$IP_t = a + \beta_1^g\, t + \beta_2^g\, t^2 + gap_t$$

for the aforementioned sample period. [2] Cooper and Priestley (2009) do not provide a theory behind the *gap* factor but show that *gap* has an excellent predictive power of future returns empirically.

4.1 Summary statistics

Panel A of Table 2 presents the correlations between the macroeconomic variables used in our estimation and the three estimated latent variables and the correlations between the latent variables and the *cay* factor and the *gap* factor. There are three main observations: (1) Both neutral technology shocks NT and investment-specific technology shock INV are positively correlated with output growth, consumption growth, and investment growth but negatively correlated with inflation. This result is consistent with our intuition and what CTW find because higher productivity leads to higher contemporaneous output, consumption, and investment. (2) NT is positively correlated with wage while the correlation between NT is close to zero and negative. Neutral technology shock improves the productivity of labor hence the wage rate. Investment-

[2] Please see Cooper and Priestley (2009) for details.

specific technology shocks also improves the productivity of labor due to high capital level, but can decrease the demand for labor and generates a downward pressure on wage rate. The final effect depends on which of the aforementioned two effects dominates. (3) Both neutral technology shock and investment-specific technology shock have a positive (although small) correlation with interest rate. Higher productivity leads to higher output, which through Taylor rule results in a higher interest rate.

The *cay* factor is positively correlated with the investment-specific technology shock and monetary policy shock with correlation coefficients being 0.64 and 0.15, and slightly negatively correlated with the neutral technology shock with a correlation coefficient of -0.02. The *gap* factor is positively correlated monetary policy shock and negatively correlated with the two technology shocks, although the magnitudes of the correlation coefficients are small, being -0.05, -0.11, and 0.08, respectively. It is possible that the *gap* captures some other fundamental shocks not included in our model.

4.2 Predictive regressions

We follow Lettau and Ludvigson (2001) and Cooper and Priestley (2009) and use the following predictive regression

$$R_{t+\tilde{1}} = \alpha + \beta\, X_t + \epsilon_t\,, \tag{17}$$

where α is the regression intercept, β is the coefficient vector, and X_t is the vector of latent shocks of quarter t, i..e,

$$X_t = [\mu_t^z,\, \mu_t^\psi,\, V_t]$$

and *cay* or *gap* for comparison reason. Here, $R_{t+\tilde{1}}$ is the excess return of CRSP value-weighted returns of the subsequent month, the subsequent quarter, or the subsequent year of quarter t for predictive regression at one-month horizon, one-quarter horizon, or one-year horizon, respectively. Therefore, $\tilde{1}$ refers to one month, one quarter, or one year accordingly.

Table 3 reports the regression coefficients using the latent variables as independent variables at three horizons, one-month, one-quarter, and one-year horizon. Panel A presents the results for the full sample period: 1966Q1 - 2010Q3. Panels B and C presents the results for two subsamples: 1970Q1-2010Q3 and 1975Q1 and 2010Q3. The choices of subsamples are chosen based on the observation in Welch and Goyal (2009) that most of successful predictors for stock returns are found to perform much worse in these two subsamples. The reported t-statistics are corrected for heteroskedasticity and serial correlation, up to two lags, using the Newey and West (1987) estimator.

The main observation from Table 3 is that our estimated latent shocks preforms well in all three sample periods with the adjusted R-squares ranging from 0.02 to 0.03 for one-month horizon, from 0.02 to 0.04 for one-quarter horizon, and from 0.7 to 0.13 for one-year horizon. The sample period 1975Q1-2010Q3 perform the worst. Most of the predictive power comes from the neutral technology shocks and the monetary policy shocks. The regression coefficients of NT and MP are almost all significant at 5% level. The explanatory power of the investment-specific technology shock is weak at one-month horizon, gets stronger at one-quarter horizon and becomes significant at 5% level at one-year horizon. This observation is present in all there sample periods. Consistent at all horizons and all sample periods, a positive neutral technology shock and a higher investment-specific technology shock lead to higher future stock returns, while higher monetary policy shocks leads to lower future stock returns. It is intuitive that higher technology level leads to higher profitability hence higher returns. The negative relation between monetary policy shocks and stock returns are consistent with the findings in Bernanke and Kuttner (2005) and may be explained by the higher financing cost of firms after a positive monetary policy shock.

We also compare the predictability of our estimated latent shocks with two of the most successful stock return predictors in the literature, the *cay* factor and the *gap* factor. Besides the success of *cay* and *gap* in predicting returns, we choose those two factors because they are constructed based on macroeconomic variables in stead of prices, such as dividend-to-price ratio. Table 4 reports the regression coefficients, the corresponding Newey-West t-statistics, and the adjusted R-squares of the estimated latent shocks, the *cay* factor, and the *gap* factor at one-month horizon. For all three sample periods, both *cay* and *gap* have a (adjusted) R-square of zero and either the coefficients of *cay* or those of *gap* are significant at 5% level. The latent shocks have a R-square ranging from 0.02 to 0.03. Monetary policy shock has the best explanatory power among the three latent variables, whose regression coefficient is significant at 5% level for all three periods. The coefficient of NT is significant at 5% level for period 1975Q1-2010Q3 and only significant at 10% level for periods 1966Q1 - 2010Q3 and 1070Q1 - 2010Q3. The coefficient of INV is not significant for all three periods. The period of 1975Q1 - 2010Q3 is the most unpredictable period for all the predictors at one-month horizon.

Table 5 compares the predictability of the latent shocks, the *cay* factor, and the *gap* factor at one-quarter horizon. The latent shocks still have the best predictive power at one-quarter horizon. The R-squares of *cay* and *gap* are lower than those of the latent shocks for all three sample periods. The coefficients of *cay* and *gap* are significant at 5% level for all three periods. Moreover, higher *cay* predicts higher future stock returns while higher *gap* predicts lower future returns. The coefficient of INV remains insignificant. The coefficients of NT and MP are significant at 5% level for all periods except that the coefficient of MP is not significant for period 1975Q1 - 2010Q3. For all predictors, period 1975Q1 - 2010Q3 remains to be the most unpredictable period.

Table 6 compares the predictability of the latent shocks, the *cay* factor, and the *gap* factor at one-year horizon. Similar to the observation at one-month and one-quarter horizons, our estimated latent variables have a better predictability than *cay* and *gap*. The R-squares of latent shocks range from 0.05 to 0.10. The coefficients of *cay* and *gap* range from all significant at 5% level. The coefficients of NT, INV, and MP are significant at 5% level except the coefficient of NT for period 1966Q1 - 2010Q3.

In summary, our estimated latent shocks has a predictive power that is to the least not worse than *cay* and *gap*. The relation between the three shocks and future returns are economically intuitive. Higher neutral technology shocks and higher investment-specific technology shocks means higher profits in the future hence higher return. Higher monetary policy shocks predict lower future returns due to higher financing costs for firms. Consistent with the positive and high correlation between *cay* and NT shown in Table 2, the *cay* factor has a similar relation with stock returns as NT. However, the economic intuition behind *gap* is hard to interpret. *gap* has very low correlation with any of the three latent shocks and higher *gap* predicts lower future returns.

5 Conclusion

A full-information Bayesian Markov Chain Monte Carlo (BMCMC) method is developed for estimating DSGE models using macroeconomic variables. We implement this method on a standard medium-size DSGE model based on CTW and extract three exogenous latent shocks: neutral technology shock, investment-specific technology shock, and monetary policy shock. The estimated latent shocks are shown to exhibit excellent predictive power for future aggregate stocks returns at one-month, one-quarter, and one-year horizon for all three sample periods examined in the study: 1966Q1-2010Q3, 1970Q1-2010Q3, and 1975Q1-2010Q3. Compared with *cay* and *gap*, our estimated latent shocks have greater and more robust predictive power.

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Table 1: **Estimated Parameters and Estimation Errors**

Panel A of this table reports the parameters values estimated using the Bayesian Markov Chain Monte Carlo method based on 50,000 Monte Carlo iterations. Observed macroeconomic variables used in the estimation are output growth (dy), consumption growth(dc), investment growth (di), wage growth (dw), logarithm of inflation (π), 3-month T-Bill (r), and employment (h). Sample period is 1966Q1 - 2010Q3. Panel B of this table reports the estimation errors of the seven observed macroeconomic variables.

Parameter	Posterior Mean	Posterior Standard Error
Panel A: Estimated parameter values		
β	0.9980	0.0005
ϕ	1.3838	0.0366
b	0.9598	0.0111
α	0.2308	0.0006
ξ_p	0.6022	0.0064
ξ_w	0.8232	0.0029
λ_f	1.1640	0.0024
λ_w	1.0373	0.0008
σ_a	0.2463	0.0242
σ_s	4.6910	0.2075
π_{ss}	1.0071	0.0023
ρ_R	0.7947	0.0036
ρ_π	1.6597	0.0524
ρ_y	0.1505	0.0079
μ_z	0.0038	0.0001
μ_ψ	0.0025	0.0003
ρ_z	0.1207	0.0664
ρ_ψ	0.7455	0.0500
ρ_v	0.3101	0.0534
σ_z	0.0026	0.0004
σ_ψ	0.0029	0.0003
σ_v	0.0021	0.0001
Panel B: Estimation errors		
σ_{dy}	0.8277	0.0446
σ_{dc}	0.4954	0.0268
σ_{di}	3.2050	0.1695
σ_{dw}	0.6202	0.0341
σ_π	0.1702	0.0161
σ_r	0.0817	0.0115
σ_h	2.9769	0.1874

Table 2: **Correlation Matrix**

Panel A of this table presents the correlations between macroeconomic variables, including per capita output growth (dy), per capita consumption growth(dc), per capita investment growth (di), wage growth (dw), logarithm of inflation (π), 3-month T-Bill (r), and average weekly hours per capita(h), and estimated latent variables, including the neutral technology shock (NT), investment-specific technology shock (INV), monetary policy shock (MP), and the *cay* factor (Lettau and Ludvigson, 2001), and output gap (Cooper and Priestley, 2009). The *cay* factor is taken from Sidney Ludvigson's website. The output *gap* factor is the residual μ_t from the following regression over the 1966Q01 and 2010Q03 sample: $y_t = a + b \cdot t + c \cdot t^2 + \mu_t$, where y_t is the log of industrial production at quarter t. Panels B of this table reports the correlation matrix between latent variables. All data are sampled quarterly from 1966Q1 to 2010Q3.

	Panel A: Correlations between macroeconomic variables and latent variables		
	NT	INV	MP
dy	0.31	0.25	0.38
dc	0.21	0.21	0.31
di	0.25	0.18	0.30
dw	0.21	-0.03	0.03
pi	-0.31	-0.32	-0.06
r	0.04	0.04	0.48
h	0.03	0.37	0.24
cay	-0.02	0.64	0.15
gap	-0.05	-0.11	0.08
	Panel B: Correlations between latent variables		
NT	1.00		
INV	0.07	1.00	
MP	0.37	0.39	1.00

Table 3: **Return Predictability of Estimated Latent Variables**

This table reports the results from predictive regressions of stock market returns on lagged latent variables. Stock market returns are proxed by the excess returns of the CRSP value-weighted index, taken from Ken French's website. Panels A, B and C present the results for three data periods, 1966Q1-2010Q3, 1970Q1-2010Q3, and 1975Q1-2010Q3, respectively. Within each panel, the results of an OLS regression where quarterly latent variables predict stock returns of one month ahead, one quarter ahead, and one year ahead are reported, respectively. The Newey-West corrected t-statistics are reported in parentheses and $\bar{R}^2$ is the adjusted R^2.

constant	NT	INV	MP	$\bar{R}^2$
		Panel A: 1966Q1 - 2010Q3		
		One-month horizon		
-1.24	302.12	37.20	-547.52	0.03
(-1.46)	(1.79)	(0.60)	(-2.24)	
		One-quarter horizon		
-0.04	9.33	1.78	-8.67	0.04
(-2.14)	(3.04)	(1.48)	(-2.23)	
		One-year horizon		
-0.03	6.62	11.96	-24.58	0.11
(-0.61)	(0.81)	(3.18)	(-2.72)	
		Panel B: 1970Q1 - 2010Q3		
		One-month horizon		
-1.44	326.26	39.32	-558.14	0.03
(-1.57)	(1.81)	(0.62)	(-2.14)	
		One-quarter horizon		
-0.04	9.94	1.77	-8.69	0.04
(-2.07)	(3.00)	(1.43)	(-2.10)	
		One-year horizon		
-0.07	17.41	11.17	-28.94	0.13
(-1.65)	(2.50)	(2.88)	(-3.79)	
		Panel C: 1975Q1 - 2010Q3		
		One-month horizon		
-1.37	364.09	34.31	-454.57	0.02
(-1.43)	(2.01)	(0.50)	(-1.96)	
		One-quarter horizon		
-0.02	8.39	0.81	-6.36	0.02
(-1.31)	(2.56)	(0.65)	(-1.44)	
		One-year horizon		
-0.05	18.09	8.81	-21.25	0.07
(-1.18)	(2.55)	(2.09)	(-2.47)	

Table 4: **Return Predictability Comparison at One-Month Horizon**

This table reports the results from predictive regressions of stock market returns at one-month horizon on lagged latent variables, the *cay* factor (Lettau and Ludvigson, 2001), and output gap (Cooper and Priestley, 2009). The *cay* factor is taken from Sidney Ludvigson's website. The output gap is the residual μ_t from the following regression over the 1966m01 and 2010m09 sample: $y_t = a + b \cdot t + c \cdot t^2 + \mu_t$, where y_t is the log of industrial production at month t. Panels A, B and C present the results for three data periods, 1966Q1-2010Q3, 1970Q1-2005Q4, and 1975Q1-2005Q4, respectively. Within each panel, the results of an OLS regression where quarterly latent shocks, the *cay* factor, and the *gap*, predict stock returns of one month ahead, respectively. The Newey-West corrected t-statistics are reported in parentheses and $\bar{R}^2$ is the adjusted R^2.

constant	*NT*	*INV*	*MP*	*cay*	*gap*	$\bar{R}^2$
			Panel A: 1966Q1 - 2010Q3			
-1.24	302.12	37.20	-547.52			0.03
(-1.46)	(1.79)	(0.60)	(-2.24)			
0.48				0.28		0.00
(1.28)				(1.85)		
0.49					-8.92	0.00
(1.31)					(-1.56)	
			Panel B: 1970Q1 - 2010Q3			
-1.44	326.26	39.32	-558.14			0.03
(-1.57)	(1.81)	(0.62)	(-2.14)			
0.37				0.29		0.00
(0.96)				(1.89)		
0.41					-8.80	0.00
(1.04)					(-1.52)	
			Panel C: 1975Q1 - 2010Q3			
-1.37	364.09	34.31	-454.57			0.02
(-1.43)	(2.01)	(0.50)	(-1.96)			
0.51				0.21		-0.00
(1.2)				(1.29)		
0.55					-8.03	0.00
(1.35)					(-1.32)	

Table 5: **Return Predictability Comparison at One-Quarter Horizon**

This table reports the results from predictive regressions of stock market returns at one-month horizon on lagged latent variables, the *cay* factor (Lettau and Ludvigson, 2001), and output gap (Cooper and Priestley, 2009). The *cay* factor is taken from Sidney Ludvigson's website. The output gap is the residual μ_t from the following regression over the 1966m01 and 2010m09 sample: $y_t = a + b \cdot t + c \cdot t^2 + \mu_t$, where y_t is the log of industrial production at month t . Panels A, B and C present the results for three data periods, 1966Q1-2010Q3, 1970Q1-2005Q4, and 1975Q1-2005Q4, respectively. Within each panel, the results of an OLS regression where quarterly latent shocks, the *cay* factor, and the *gap*, predict stock returns of one quarter ahead, respectively. The Newey-West corrected t-statistics are reported in parentheses and $\bar{R}^2$ is the adjusted R^2.

constant	NT	INV	MP	*cay*	*gap*	$\bar{R}^2$
			Panel A: 1966Q1 - 2010Q3			
-0.04	9.33	1.78	-8.67			0.04
(-2.14)	(3.04)	(1.48)	(-2.23)			
0.01				0.01		0.02
(1.87)				(2.44)		
0.01					-0.29	0.03
(2.03)					(-2.52)	
			Panel B: 1970Q1 - 2010Q3			
-0.04	9.94	1.77	-8.69			0.04
(-2.07)	(3.00)	(1.43)	(-2.10)			
0.01				0.01		0.02
(1.80)				(2.41)		
0.01					-0.28	0.03
(2.03)					(-2.45)	
			Panel C: 1975Q1 - 2010Q3			
-0.02	8.39	0.81	-6.36			0.02
(-1.31)	(2.56)	(0.65)	(-1.44)			
0.02				0.01		0.01
(2.19)				(1.68)		
0.02					-0.20	0.01
(2.53)					(-1.66)	

Table 6: **Return Predictability Comparison at One-Year Horizon**

This table reports the results from predictive regressions of stock market returns at one-month horizon on lagged latent variables, the *cay* factor (Lettau and Ludvigson, 2001), and output gap (Cooper and Priestley, 2009). The *cay* factor is taken from Sidney Ludvigson's website. The output gap is the residual μ_t from the following regression over the 1966m01 and 2010m09 sample: $y_t = a + b \cdot t + c \cdot t^2 + \mu_t$, where y_t is the log of industrial production at month t . Panels A, B and C present the results for three data periods, 1966Q1-2010Q3, 1970Q1-2005Q4, and 1975Q1-2005Q4, respectively. Within each panel, the results of an OLS regression where quarterly latent shocks, the *cay* factor, and the *gap*, predict stock returns of one year ahead, respectively. The Newey-West corrected t-statistics are reported in parentheses and $\bar{R}^2$ is the adjusted R^2.

constant	NT	INV	MP	*cay*	*gap*	$\bar{R}^2$
			Panel A: 1966Q1 - 2010Q3			
-0.03	6.62	11.96	-24.58			0.11
(-0.61)	(0.81)	(3.18)	(-2.72)			
0.05				0.03		0.10
(2.61)				(3.35)		
0.06					-1.02	0.09
(3.05)					(-2.90)	
			Panel B: 1970Q1 - 2010Q3			
-0.07	17.41	11.17	-28.94			0.13
(-1.65)	(2.50)	(2.88)	(-3.79)			
0.06				0.03		0.10
(2.56)				(3.10)		
0.06					-0.96	0.08
(3.10)					(-2.71)	
			Panel C: 1975Q1 - 2010Q3			
-0.05	18.09	8.81	-21.25			0.07
(-1.18)	(2.55)	(2.09)	(-2.47)			
0.06				0.02		0.05
(2.66)				(2.26)		
0.07					-0.72	0.05
(3.37)					(-2.01)	

Figure 1: **Estimated Latent Variables**

This figure plots the estimated latent variables: total factor productivity μ_z, investment-specific technology μ_ψ, and monetary policy shock μ_V for sample period 1966Q1 - 2010Q3.