An Empirical Comparison of Non-traded and Traded Factors in Asset Pricing[†]

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Abstract

In this paper, we argue that it is advisable to use traded factors rather than their non-traded counterparts from the econometric perspective, because non-traded factors are found to be weakly correlated with asset returns, which makes statistical findings in the Fama-MacBeth two-pass procedure unreliable. To illustrate the weak correlation between non-traded factors and asset returns and its implied inference problem on risk premium, we adopt three methods. We first use the method of Bai and Ng (2006), and find that many non-traded factors are only weakly related to the latent factors and thus asset returns, which is further confirmed by our second adopted method, the rank test of Kleibergen and Paap (2006); in contrast, traded counterparts of non-traded factors are found to be more closely related to latent factors as well as asset returns. Finally, as a third method, we invert the factor statistics in Kleibergen (2009) to construct confidence intervals of risk premium associated with these factors, and find that non-traded factors seem to be less informative for deriving risk premium than traded factors, which also serves as the indirect evidence for the weak statistical quality of non-traded factors.

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1 Introduction

The sizeable literature of asset pricing has suggested a large group of macroeconomic factors that seem to capture the systematic risk and help explain the return of financial assets. Examples of these factors include the residential and nonresidential investment growth in Cochrane (1996), the durable and nondurable consumption growth in Yogo (2006), the investment growth in household, financial and nonfinancial business in Li et al. (2006), the funding liquidity in Muir et al. (2011), among many others.

To evaluate the validity of these proposed risk factors in asset pricing, the most widely used approach is the Fama and MacBeth (1973) (FM) two-pass procedure with the Shanken (1992) correction, see, e.g., Kan and Robotti (2012) for a survey. However, Kan and Zhang (1999) show that when the proposed factors are completely useless, the FM two-pass procedure may yield spurious empirical results that seem to favor these useless factors. Kleibergen (2009) further highlights that if the proposed factors are not useless but only weakly correlated with asset returns, the FM two-pass procedure is similarly jeopardized: specifically, when factors are weak, statistical findings of the FM two-pass procedure in empirical asset pricing studies (e.g. t-statistic of risk premium) are unreliable. Although non-traded macroeconomic factors proposed in the asset pricing literature may not be completely useless, we show in this paper that their correlation with latent factors and asset returns is very weak, based on the evidence from Bai and Ng (2006)'s regression approach and Kleibergen and Paap (2006)'s rank test. Consequently, this paper casts doubt on the seeming success of these non-traded macroeconomic factors in the FM two-pass procedure. On the other hand, the asset return based traded factors, which are the counterparts of the non-traded macroeconomic factors, are found to be more closely related to latent factors as well as asset returns, and their associated confidence intervals of risk premium constructed by inverting Kleibergen (2009)'s factor statistics are often more informative, hence this paper recommends using traded factors to replace their non-traded counterparts in empirical applications.

In this paper, we first adopt the approach of Bai and Ng (2006) to show that the non-traded macroeconomic factors proposed in the asset pricing literature are weakly correlated with the latent risk factors in a linear factor model, which further suggests that the correlation between these factors and asset returns is also weak. The idea of this approach can be described as follows. We start by estimating the space of the unobservable latent factors in a linear factor model for asset returns, then continue to examine whether the proposed factors are related to the estimated latent factors. If we find the proposed risk factors to be closely related to the latent factors, then it provides evidence to support these proposed factors; if in contrast, we find that the proposed factors are not statistically related to the latent factors, then it provides evidence that these proposed factors may be useless or weak, i.e. nearly useless.

Bai and Ng (2006) propose the approach described above to evaluate the proposed risk factors. However, the empirical work that has applied Bai and Ng (2006)'s approach to the asset pricing literature is limited. We think that the reason could be two-fold. Firstly, the pitfalls of the popular FM two-pass procedure were not fully discussed until the recent work of Kleibergen (2009) and Lewellen et al. (2010), hence the demand for a novel methodology was not urgent in this literature. Secondly, Bai and Ng (2006)'s approach requires a large number of financial assets, which is not always satisfied in practice. For example, the Fama-French 25 size and book-to-market sorted portfolios are commonly used as the test portfolios in the empirical studies of asset pricing, and 25 is a relatively small sample size, which jeopardizes the empirical applications of Bai and Ng (2006) in the asset pricing area[1].

Following the suggestion of Lewellen et al. (2010), we augment the conventional size and book-to-market sorted portfolios with the industry portfolios to construct the set of test portfolios in this paper. The sample size of the test portfolios thus increases to a level that now suits Bai and Ng (2006). We then use these portfolios to estimate the latent factors by the principal component analysis (PCA) explained in Connor and Korajczyk (1988) and Bai and Ng (2002), and proceed to examine whether the macroeconomic factors proposed by the aforementioned papers are related to the latent ones, by regressing the macroeconomic factors on the principal components. In the empirical application, we find that many of the non-traded macroeconomic factors are not strongly related to the latent factors. These findings raise the concern that β, the correlation matrix of asset returns and proposed factors, has small magnitude, which further implies the seeming success of the proposed factors in the FM two-pass procedure is under doubt, due to the reasons discussed in Kleibergen (2009), Kleibergen and Zhan (2013).

[1]When the sample size is larger, the methodology of Bai and Ng (2006) has been applied in various settings, e.g. Goyal et al. (2008) and Lin et al. (2012), the focus of which, however, are not on evaluating the macroeconomic factors proposed in the asset pricing literature.

In contrast, the traded counterparts of these macroeconomic factors constructed by asset returns following the instruction of Fama and French (1993) and Pástor and Stambaugh (2003), are found to be more closely related to the latent factors, which further suggests statistical inference with traded factors in the second pass of the FM two-pass procedure is more credible than that with non-traded factors.

Apart from showing that the correlation of non-traded macroeconomic factors and latent factors seems to be weaker than that of traded factors and latent factors, we further compare non-traded factors and their traded counterparts in another two ways. On one hand, we apply the Kleibergen and Paap (2006) rank test to β, in order to examine whether the proposed factors are closely related to asset returns, and we find that compared to their non-traded counterparts, traded factors are more closely related to asset returns, which is consistent with the findings when we apply the methodology of Bai and Ng (2006). On the other hand, by employing the factor statistics in Kleibergen (2009) to construct the trustworthy 95% confidence intervals (C.I.'s) of risk premium associated with both non-traded and traded factors, we find that traded factors tend to have bounded C.I.'s of risk premium, while the C.I.'s associated with non-traded factors are often unbounded, which indicates that traded factors are more informative than their non-traded counterparts. We also report that C.I.'s of risk premium constructed by Kleibergen (2009)'s factor statistics, the validity of which does not depend on the quality of proposed factors, are often substantially different from C.I.'s constructed by the FM t-statistic, which requires that the proposed factors are good proxies for latent factors. The substantial difference between C.I.'s of risk premium constructed by factor statistics and C.I.'s by FM t-statistic further reflects the doubtful quality of macroeconomic factors for asset pricing.

The rest of the paper is organized as follows. The linear factor model and the empirical findings based on Bai and Ng (2006)'s approach are presented in Section 2. In Section 3, we adopt the rank test of Kleibergen and Paap (2006) to detect possibly weak or useless factors, which also serves to double-check our findings in Section 2. In Section 4, C.I.'s of risk premium associated with both non-traded and traded factors are constructed for comparison, by inverting Kleibergen (2009)'s factor statistics. Section 5 concludes.

2 Relating Proposed Factors to Latent Factors

2.1 Preliminary

2.1.1 Linear Factor Model and Principal Component Analysis

The FM two-pass procedure involves a linear factor model for financial asset returns, in which the excess return of asset i at time t, denoted by R_{it}, is linearly related to k latent factors $F^*_{1t}, .., F^*_{kt}$:

$$R_{it} = \beta^*_{i1} F^*_{1t} + ... + \beta^*_{ik} F^*_{kt} + e_{it} \tag{1}$$

where $i = 1, ..., N$, $t = 1, ..., T$, e_{it} is the idiosyncratic error unrelated to the latent factors according to the arbitrage pricing theory in Ross (1976).

The model can be rewritten as:

$$R_{it} = \beta^{*\prime}_i F^*_t + e_{it} \tag{2}$$

with $\beta^*_i = (\beta^*_{i1}, ..., \beta^*_{ik})'$, $F^*_t = (F^*_{1t}, ..., F^*_{kt})'$. Furthermore, if we define two $N \times 1$ vectors $R_t = (R_{1t}, ..., R_{Nt})'$ and $e_t = (e_{1t}, ..., e_{Nt})'$, as well as an $N \times k$ full rank matrix $\beta^* = (\beta^*_1, ..., \beta^*_N)'$, then we have:

$$R_t = \beta^* F^*_t + e_t \tag{3}$$

which coincides with Equation (1) in Goyal et al. (2008) and Lewellen et al. (2010).

The k latent factors denoted by F^*_t in the linear factor model above are unobservable in practice. Instead, researchers may propose m observable risk factors, denoted by the $m \times 1$ vector F_t, as the proxy for F^*_t. Note that m is not necessarily equal to k. This is corresponding to the fact that many different versions of asset pricing models have been proposed in the past decades, and the number of factors in these models varies. See, e.g., Fama and French (1993), Acharya and Pedersen (2005), Yogo (2006) and Muir et al. (2011).

If F_t can serve as the good proxy for the latent F^*_t, then the elements in F_t are believed to be close to the elements in F^*_t, or more generally, the elements in F_t need to be at least close to some linear combination of the elements in F^*_t. Now a question naturally arises: how can we evaluate whether it is valid to consider F_t as the good proxy for F_t^* from the statistical point of view? Since F_t is observable while F_t^* is not, it is infeasible for us to directly examine the relationship between F_t and F_t^*.

In the empirical literature of asset pricing, the method commonly adopted for evaluating the validity of the risk factors in F_t is the FM two-pass procedure, which could be misleading as shown by Kan and Zhang (1999), Kleibergen (2009) and Lewellen et al. (2010). Instead of the FM two-pass procedure, this paper is intended to apply the methodology of Bai and Ng (2006) and evaluate the validity of F_t, which denotes the proposed non-traded or traded risk factors, by taking the following steps:

1^{st} step. Construct the principal components (denoted by $\tilde{F}_t^*$) for asset returns by PCA shown in, e.g., Connor and Korajczyk (1988), Bai and Ng (2002)(2006).

2^{nd} step. Examine whether the proposed factors in F_t are related to the computed principal components $\tilde{F}_t^*$, instead of directly examining whether the factors in F_t are related to the latent F_t^*.

As described in Bai and Ng (2006), under the necessary normalization, the computation of $\tilde{F}_t^*$ by PCA is straightforward: if we choose a k-factor model[2], then $\tilde{F}_t^*$ equals $\sqrt{T}$ times the eigenvectors corresponding to the k largest eigenvalues of $\frac{RR'}{NT}$, where $R = (R_1, ..., R_T)'$. Note that $\tilde{F}_t^*$ computed in the manner of Bai and Ng (2006) corresponds to the principal components in Connor and Korajczyk (1988) scaled by $\sqrt{T}$.

When the sample size is large, Bai and Ng (2006) prove that the difference between the computed $\tilde{F}_t^*$ and the latent F_t^* is negligible up to rotation. In other words, the space of $\tilde{F}_t^*$ consistently estimates the space of F_t^*. See Lemma 1 in Bai and Ng (2006). A similar result is stated in Connor and Korajczyk (1988).

The idea of Bai and Ng (2006) for evaluating the validity of the proposed risk factors can be restated as follows: if the proposed risk factors in F_t are good proxies for the latent factors in F_t^*, then F_t and F_t^* must at least be linearly related; now since the space of $\tilde{F}_t^*$ computed by PCA consistently estimates the space spanned by F_t^*, we expect to see that F_t and $\tilde{F}_t^*$ are also related. If we find the evidence that F_t and $\tilde{F}_t^*$ are not statistically related, then it is very

[2]When k is unknown, we can use some information criteria to help determine k. See, e.g. Bai and Ng (2002).

suspicious that F_t could be used as the good proxy for F_t^*; or in other words, the proposed factors in F_t are unlikely to be the ideal factors.

2.1.2 Statistics of Bai and Ng (2006)

Given $\tilde{F}_t^*$ computed by PCA and the proposed F_t, we are now ready to evaluate the relationship between the proposed risk factors and the latent factors. To do so, Bai and Ng (2006) advocate to use several test statistics, which we briefly describe below. These statistics are denoted by $A(j)$, $M(j)$, $R^2(j)$, $NS(j)$ and $\hat{\rho}(k)^2$ respectively.

Let F_{jt} be the j^{th} proposed factor in F_t, and we are interested in evaluating whether it is related to latent factors. Consider an auxiliary linear regression: regress F_{jt} on the principal components $\tilde{F}_t^*$. Let $\hat{F}_{jt}, \hat{\epsilon}_{jt}$ be the predicted value of F_{jt} and the residual based on this auxiliary regression, and construct the t-statistic:

$$\hat{\tau}_t(j) = \frac{\hat{F}_{jt} - F_{jt}}{(\widehat{Var}(\hat{F}_{jt}))^{1/2}}$$

where $\widehat{Var}(\hat{F}_{jt})$ stands for the estimated variance of $\hat{F}_{jt}$. Bai and Ng (2006) show that under the null hypothesis that F_{jt} is perfectly linearly related to latent risk factors in F_t^*, i.e. $F_{jt} = \delta_j' F_t^*$ for some time-invariant δ_j, the t-statistic $\hat{\tau}_t(j)$ converges to the standard normal distribution.

Based on the auxiliary regression and $\hat{\tau}_t(j)$ described above, Bai and Ng (2006) continue to define $A(j)$, $M(j)$, $R^2(j)$, $NS(j)$ associated with the j^{th} proposed factor F_{jt}:

i. $A(j) = \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}(|\hat{\tau}_t(j)| > \Phi_{1-\alpha})$

ii. $M(j) = max_{1 \le t \le T} |\hat{\tau}_t(j)|$

iii. $R^2(j) = \frac{\widehat{Var}(\hat{F}_{jt})}{\widehat{Var}(F_{jt})}$

iv. $NS(j) = \frac{\widehat{Var}(\hat{\epsilon}_{jt})}{\widehat{Var}(\hat{F}_{jt})}$

where $\Phi_{1-\alpha}$ is the $1-\alpha$ quantile of the standard normal distribution, $\widehat{Var}(\hat{\epsilon}_{jt})$ and $\widehat{Var}(F_{jt})$ are the estimated variance[3] of $\hat{\epsilon}_{jt}$ and F_{jt}.

[3]For the exact formula of $\widehat{Var}(\hat{F}_{jt})$, $\widehat{Var}(\hat{\epsilon}_{jt})$ and $\widehat{Var}(F_{jt})$, see Bai and Ng (2006).

As N, $T \to \infty$ with[4] $\sqrt{N}/T \to 0$, Bai and Ng (2006) show the following results: if F_{jt} is perfectly linearly related to F_t^*, then $A(j) \xrightarrow{p} 2\alpha$ and $M(j)$ has a non-standard asymptotic distribution whose critical values are tabulated in Bai and Ng (2006); if F_{jt} is not perfectly linearly related to F_t^*, but fairly close to a linear combination of the elements in F_t^*, then $R^2(j)$ is expected to be close to 1, while $NS(j)$ is expected to be close to 0.

Note that the four statistics above, $A(j)$, $M(j)$, $R^2(j)$ and $NS(j)$, are intended to examine whether the j^{th} element in F_t is related to latent factors. To jointly evaluate the relationship between the proposed and latent factors, Bai and Ng (2006) suggest to report the canonical correlations of $\tilde{F}_t^*$ and F_t, denoted by $\hat{\rho}(1)^2, \hat{\rho}(2)^2, ...$, where:

v. $\hat{\rho}(k)^2 =$ the k^{th} largest eigenvalue of the matrix $S^{-1}_{\tilde{F}^*\tilde{F}^*} S_{\tilde{F}^*F} S^{-1}_{FF} S_{F\tilde{F}^*}$, where S_{AB} stands for the estimated covariance matrix between A and B.

If the proposed factors in F_t coincide with the latent factors, then we expect that all the non-zero canonical correlations are close to 1. Furthermore, Bai and Ng (2006) derive the asymptotic distribution of $\hat{\rho}(k)^2$, which is useful for constructing the confidence intervals of $\hat{\rho}(k)^2$ and $R^2(j)$, since in the special case that $m = 1$, $\hat{\rho}(1)^2 = R^2(j)$.

To summarize, the i-v statistics suggested by Bai and Ng (2006) will be used in this paper to evaluate the relationship between the proposed risk factors and the latent factors: $A(j)$ and $M(j)$ are used to test if the proposed single j^{th} factor F_{jt} is perfectly linearly related to the latent F_t^*, while $R^2(j)$ and $NS(j)$ are used to evaluate whether F_{jt} is not equal but close to a linear combination of the latent factors in F_t^*; in addition, $\hat{\rho}(k)^2$ measures the joint relationship between the proposed F_t and the latent F_t^*.

2.2 Application

2.2.1 Data Description

The portfolios used in our empirical application are downloaded from Kenneth French's web site[5]. Following Lewellen et al. (2010), we use two types of portfolios, the size and book-to-market sorted portfolios and the industry portfolios. We augment the conventional size

[4]In our application, N stands for the number of test assets, T stands for the number of time periods, whose values are appropriate to meet the conditions in Bai and Ng (2006).

[5]*http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.*

and book-to-market sorted portfolios with the industry portfolios to construct the set of test portfolios.

Specifically, we combine the 100 size and book-to-market sorted portfolios and the 49 industry portfolios to construct a test set. We choose the 100 and 49 portfolios because they contain the largest numbers of portfolios in their categories, so the sample size is large enough to apply Bai and Ng (2006)'s methodology. For the purpose of robustness check, we similarly combine the 25 size and book-to-market sorted portfolios and the 30 industry portfolios as in Lewellen et al. (2010) to construct an alternative test set. Although we get the monthly return data for these portfolios, we also convert the monthly return to the quarterly return, because the non-traded macroeconomic factors are typically quarterly available. The risk free return is the treasury bill rate and the excess return is constructed by subtracting the risk free return from the return.

The macroeconomic factors considered in our application include the residential investment growth $\triangle I_{Res}$ and nonresidential investment growth $\triangle I_{Nres}$ in Cochrane (1996), the durable consumption growth $\triangle C_{Dur}$ and the nondurable consumption growth $\triangle C_{Ndur}$ in Yogo (2006), the investment growth rate in the financial cooperations $Finan$, the nonfinancial corporate business $Nfinco$, and the household sector $Hholds$ in Li et al. (2006), the funding liquidity Lev in Muir et al. (2011). The data of these risk factors are either provided by the authors, or constructed following the descriptions in their paper. We update the data to the year 2010.

We also use the non-traded macroeconomic factors above to construct factor mimicking portfolios, which are the traded portfolios that mimic these non-traded macroeconomic factors. In particular, we follow Fama and French (1993) and Pástor and Stambaugh (2003) to form portfolios based on pre-ranking covariance of the excess return and the non-traded factors using the past 5-year rolling window. We form five equal weighted portfolios[6], then re-balance them every quarter. The constructed traded macroeconomic factor is the difference in return between the two portfolios with the lowest and highest covariance. The stock return data between January 1952 and December 2010 from CRSP is used in this construction. We focus on the companies listed in NYSE, AMEX and NASDAQ, whose common stocks with share codes of 10 or 11 are included.

[6]The results of forming 2, 4, 6, 8 and 10 portfolios are similar and our results are qualitatively unchanged when we use value-weighted portfolios.

For the non-traded macroeconomic factors, we use the quarterly data between 1952Q2 and 2010Q4, during which most of these factors have data available, hence $T = 235$. Portfolios with missing values during this period are removed in order to apply the methodology of Bai and Ng (2006), which requires the balance of the panel: in total, 18 portfolios in the set of 100 size and book-to-market sorted portfolios and 49 industry portfolios are removed, so we have $N = 131$ portfolios left; for the alternative test set, we keep all 55 portfolios, since no missing values are found in this set. For the traded macroeconomic factors, we use the monthly data between January 1961 and December 2010, hence $T = 600$, and we similarly use the 131 portfolios as the main test set. We have limited observations for the funding liquidity Lev in Muir et al. (2011): in particular, we only have the quarterly data in 1968Q1-2009Q4 for its non-traded version, and monthly data in 1973M1-2009M12 for its traded version.

The summary statistics and correlation of the data are presented in Table 1 and 2.

2.2.2 Relating Non-traded Factors to Latent Factors

As described above, in order to evaluate the non-traded macroeconomic factors, we use the set of test portfolios made of the 100 Fama-French size and book-to-market sorted portfolios and the 49 industry portfolios between 1952Q2 and 2010Q4, and we consider in total eight macroeconomic factors from the empirical asset pricing literature. We compute all the statistics illustrated in Section 2.1.2 to evaluate the relationship between the latent factors in the test portfolios and the proposed non-traded macroeconomic factors. The empirical findings are presented in Table 3.

Since the number of the latent factors k in the test portfolios is unknown, we use the information criteria in Bai and Ng (2002) to choose it, and find that $k = 5$ or 6 is preferred[7].

If we set $k = 5$, i.e. there are five latent factors in the test portfolios, then the empirical findings are presented in the top panel of Table 3. The values of $A(j)$ show that none of the eight non-traded macroeconomic factors seem to be perfectly related to the five latent factors, since $A(j)$'s are far from the nominal 5%. Similarly, $M(j)$'s are larger than its 5% critical value 3.656 tabulated in Bai and Ng (2006), hence we reject the null hypothesis that any of the eight

[7]Specifically, this paper uses four criteria $PC(k)_{p1}$, $PC(k)_{p2}$, $IC(k)_{p1}$, $IC(k)_{p2}$ in Bai and Ng (2002). Details of these criteria can be found in their paper. $PC(k)_{p1}$, $PC(k)_{p2}$ and $IC(k)_{p1}$ choose $k = 6$, while $IC(k)_{p2}$ chooses $k = 5$.

proposed non-traded factors is a linear combination of the five latent factors. Furthermore, when we regress each of the eight non-traded macroeconomic factors on the five latent factors, we find that the corresponding $R^2(j)$'s are very small and $NS(j)$'s are large, both indicating that these proposed factors are substantially different from the five latent factors. In addition, the canonical correlations denoted by $\hat{\rho}(k)^2$ are also small, which further supports the view that none of the eight proposed macroeconomic factors are closely related to the five latent ones.

To better explain that the non-traded macroeconomic factors are indeed very weakly related to the latent factors, we also conduct a Monte Carlo study: we simulate an artificial factor from the standard normal distribution, denoted by N(0,1). Since we simulate the data independent of asset returns, this artificial factor is completely irrelevant, which corresponds to the useless factor in Kan and Zhang (1999). In the same manner as we evaluate the non-traded macroeconomic factors, we also evaluate this artificial factor by relating it to the latent factors and report its $A(j)$, $M(j)$, $R^2(j)$, $NS(j)$ and $\hat{\rho}(k)^2$ in Table 3: the point values are the median of 1000 replications, while the 95% confidence intervals result from the 2.5% and 97.5% quantiles. Not very surprisingly, we find that this useless artificial factor has large values of $A(j)$, $M(j)$ to reject that it is an exact latent factor, and it has small $R^2(j)$ (which equals $\hat{\rho}(k)^2$ since N(0,1) is evaluated as a single factor) and large $NS(j)$ to suggest that it is not closely related to any linear combination of latent factors. The interesting result is that, the performance of this useless factor denoted by N(0,1) is very similar to the performance of the eight non-traded macroeconomic factors in Table 3, as their statistics are comparable. In particular, the confidence intervals of their $R^2(j)$ are not disjoint. Consequently, we could not rule out the possibility that these non-traded macroeconomic factors are in fact useless or nearly useless for asset returns from the statistical point of view.

When we change the number of latent factors by setting $k = 6$ based on the information criteria of Bai and Ng (2002), the empirical findings are presented in the bottom panel of Table 3. The findings in the bottom panel are similar to those in the top panel of Table 3 where k is set to 5. That is, for the eight non-traded macroeconomic factors, the large values of $A(j)$, $M(j)$ reject the null hypotheses that these factors are perfectly related to the six latent factors, and the small values of $R^2(j)$, $\hat{\rho}(k)^2$ and large values of $NS(j)$ further suggest that these factors are not closely related to the six latent factors. In addition, the performance of these proposed non-traded macroeconomic factors does not significantly differ from that of a useless factor denoted by N(0,1), which is independently simulated from the standard normal distribution.

Overall, no matter whether we set[8] $k = 5$ or 6, we find the consistent evidence that the non-traded macroeconomic factors under consideration do not appear statistically related to the latent factors; in particular, their performance is comparable to that of a randomly generated useless factor. These findings raise the concern that the non-traded macroeconomic factors may be very uninformative, as they are substantially different from the leading latent factors.

2.2.3 Robustness Check I (Different Portfolio)

In the application above, the test portfolios are made of the 100 size and book-to-market sorted portfolios and the 49 industry portfolios. As a robustness check, we use an alternative set of test portfolios, which are the combination of the 25 size and book-to-market sorted portfolios and the 30 industry portfolios used in Lewellen et al. (2010). We conduct the same practice as above, in order to examine whether our empirical findings will significantly change.

We similarly use Bai and Ng (2002) to determine the number of latent factors k, which suggest the existence of 8 or 9 latent factors in the alternative test set made of 25 size and book-to-market sorted portfolios and 30 industry portfolios. We consider both of these values for k. The empirical outcome of the robustness check is presented in Table 4.

The findings in Table 4 are similar to those in Table 3, no matter k is set to 8 or 9: $A(j)$'s are far from 5%, while $M(j)$'s are all above the 5% critical value 3.656, both of which indicate that none of the non-traded macroeconomic factors are perfectly related to the latent ones. Furthermore, the values of $R^2(j)$, $NS(j)$ and $\hat{\rho}(k)^2$ consistently support the view that the non-traded macroeconomic factors under consideration are only weakly related to the latent factors. Finally, the statistics associated with the artificial useless factor N(0,1) are comparable to those of the proposed non-traded macroeconomic factors.

In terms of the point values of $R^2(j)$, it appears that the proposed non-traded macroeconomic factors perform slightly better than the useless factor denoted by N(0,1) in Table 4. This should not be surprising, since the useless factor is randomly drawn from the standard normal distribution independent of asset returns. In addition, Table 4 also shows that the confidence intervals of $R^2(j)$ are overlapping with each other, thus the $R^2(j)$ associated with

[8]We also consider other values of k, e.g. $k = 3$, to check our results, which stay qualitatively similar to those reported in the paper.

the non-traded macroeconomic factors is not distinctly larger than the $R^2(j)$ associated with the useless factor. Consequently, Table 4 still conveys the message that the non-traded macroeconomic factors are not closely related to the latent factors, when we use the alternative test set.

2.2.4 Robustness Check II (Individual Stock)

It is known that different portfolio grouping procedures may significantly change asset pricing test results, and using portfolios as test assets could generate data snooping biases (see, e.g. Lewellen et al. (2010), Chordia et al. (2011)), which might make our empirical findings above less convincing. To address this issue, we use the individual stocks as test assets to examine the proposed non-traded macroeconomic factors again. In total, we use 1411 stocks from CRSP between 1991Q1 and 2010Q4. We choose these stocks because the full observations of their returns are available during this period, while stocks with missing data are dropped, in order to apply the methodology of Bai and Ng (2006).

Table 5 presents the empirical outcome when we use the individual stocks described above to replace portfolios which are used in Table 3 and 4. The information criteria in Bai and Ng (2002) suggest the number of latent factors k is 3 or 4. As a result, we also have two panels in Table 5, corresponding to $k = 3$ or 4.

Not surprisingly, we still find that the proposed non-traded macroeconomic factors have large values of $A(j)$, $M(j)$ and $NS(j)$, but small values of $R^2(j)$ and $\hat{\rho}(k)^2$; in addition, their performance is not significantly different from that of the artificial useless factor denoted by N(0,1). Consequently, it appears that many non-traded macroeconomic factors proposed in the asset pricing literature are only weakly related to the leading latent factors of asset returns, no matter whether we use the portfolios or individual stocks to derive the latent factors.

2.2.5 Relating Traded Factors to Latent Factors

In the application and robustness checks above, we have used the quarterly data for asset returns to construct test assets, and the macroeconomic factors are non-traded. However, monthly returns are more informative than quarterly returns, in addition, the asset return based traded macroeconomic factors are also commonly used in the empirical asset pricing literature, instead of non-traded macroeconomic factors, see, e.g. Pástor and Stambaugh (2003), Muir et al. (2011). Hence it is worthy to see whether traded counterparts of the non-traded macroeconomic factors are closely related to latent factors using monthly data.

For this purpose, we use the monthly portfolio returns between $1961M1$ and $2010M12$. The test set is still made of 100 size and book-to-market sorted portfolios and the 49 industry portfolios, and we use the same macroeconomic factors as described above, but now we use their traded version. Furthermore, the three Fama and French (1993) factors, namely the excess return on market (R_M), the average return on small portfolios minus the average return on big portfolios (SMB) and the average return on the value portfolios minus the average return on the growth portfolios (HML), are also added to the set of proposed risk factors to provide a benchmark.

Table 6 presents the empirical findings, for which we use the monthly portfolio returns and traded macroeconomic factors derived from non-traded macroeconomic factors following Fama and French (1993) and Pástor and Stambaugh (2003). Again, we start by employing the information criteria in Bai and Ng (2002) to determine the number of latent factors k, which suggest the existence of 6 or 7 latent factors, depending on which information criterion we use. We consider both choices of k.

As we can see from Table 6 for which traded factors are used, our empirical findings are comparable to but slightly different from those in Table 3, 4 and 5, where non-traded factors are used. In particular, all $A(j)$'s of the traded macroeconomic factors are far from 5%, and their $M(j)$'s are still above the 5% critical value 3.656, hence none of these traded factors are likely to be the exact combination of latent factors; furthermore, the small values of $R^2(j)$ and large values of $NS(j)$ associated with the traded macroeconomic factors indicate that these factors are not as closely related to the latent factors as the three Fama-French factors. However, if we compare Table 6 with Table 3-5, we also notice that the correlation of the proposed risk factors and latent factors appears to be improved if we use traded factors instead of non-traded factors: for example, if we look at the values of $R^2(j)$, it is clear that in most cases (the only exception is $Nfinco$ when $k = 6$), $R^2(j)$ gets much larger when we use traded factors in Table 6 instead of non-traded factors in Table 3-5, which indicates that traded factors are more closely related to latent factors, compared to non-traded macroeconomic factors.

In contrast with traded macroeconomic factors, although the Fama-French factors are unlikely to coincide with the exact latent factors (as their $A(j)$'s differ from 5% and $M(j)$'s exceed the critical value), these three factors all have distinctly large values of $R^2(j)$ and small values of $NS(j)$, both of which indicate that the Fama-French factors are closely related to the latent factors. Furthermore, the largest three values of $\hat{\rho}(k)^2$'s are very close to 1 while the others are not, suggesting that among all the eleven factors (three Fama-French factors plus eight traded macroeconomic factors), only three of them are closely related to the latent factors. In other words, although traded macroeconomic factors are more closely related to latent factors, compared to their non-traded counterparts, their performance is not comparable to that of the three Fama-French factors.

All the empirical findings stated above remain almost unaffected[9], no matter k is 6 or 7.

3 Rank Test for β

In the previous section, we report that many non-traded macroeconomic factors are not strongly related to the unobservable but estimable latent factors for asset returns. In addition, we also find some evidence that traded macroeconomic factors constructed based on asset returns, appear more closely related to the latent factors compared to their non-traded counterparts, which indicates that statistical inference with traded factors in the second pass of the FM two-pass procedure is more reliable than that with non-traded factors. However, the evidence is not sufficiently compelling, e.g., we use monthly traded factors but quarterly non-traded factors in Section 2, and the time periods used for non-traded and traded factors also differ, both of which might arguably have caused the difference between Table 3-5 and Table 6.

In this and the next section, we will explicitly compare the performance of non-traded macroeconomic factors and the corresponding traded factors in two ways, respectively. To do so, we limit the time period to $1973Q1-2009Q4$, during which we have quarterly data available for both non-traded and traded factors. Thus we eventually use the same frequency and the same time period for both non-traded and traded factors, for better comparison.

[9]To double-check our results in Table 3-6, we also use other commonly used portfolios in asset pricing such as 25 size and book-to-market and 25 size and momentum portfolios, and so on; in addition, the original data periods as in Cochrane (1996), Yogo (2006), Li et al. (2006) and Muir et al. (2011) are also considered. The results are qualitatively similar to those presented in Table 3-6, hence we omit them for brevity. Details of these results are available on request.

Instead of applying Bai and Ng (2006) to evaluate the proposed factors, we now adopt a more straightforward approach, the rank test on β, which denotes the correlation matrix of asset returns and proposed factors:

$$\beta \equiv cov(R_t, F_t)var(F_t)^{-1} \tag{4}$$

Note that $\beta^* = cov(R_t, F_t^*)var(F_t^*)^{-1}$ based on Equation (3), which coincides with β if $F_t = F_t^*$.

Since the FM two-pass procedure assumes that asset returns are a linear combination of latent factors and the idiosyncratic error in the linear factor model described by Equation (1)-(3), our findings in Section 2 suggest that the correlation between many recently proposed non-traded macroeconomic factors and asset returns is weak, which further implies that the corresponding β is likely to have small magnitude. Consequently, we can gauge the quality of the proposed non-traded as well as traded factors by directly examining β.

To show this (see also Kleibergen and Zhan (2013) for the derivation), we start with the following equation implied by Equation (3):

$$cov(R_t, F_t) = \beta^* cov(F_t^*, F_t) + cov(e_t, F_t) \tag{5}$$

Since the idiosyncratic error e_t is unrelated to the proposed F_t, Equation (5) reduces to:

$$cov(R_t, F_t) = \beta^* cov(F_t^*, F_t) \tag{6}$$

Hence:

$$\beta \equiv cov(R_t, F_t)var(F_t)^{-1} = \beta^* cov(F_t^*, F_t)var(F_t)^{-1} \tag{7}$$

In the extreme case that the proposed factors are completely useless, $cov(F_t^*, F_t)$ is zero, which implies $cov(R_t, F_t)$ and β also reduce to zero, thus the β matrix has reduced rank. In contrast, if F_t coincides with F_t^*, then β equals β^*, the $N \times k$ full rank matrix. Consequently, a rank test can be employed here to directly examine whether β has full rank, to help gauge whether the proposed factors are useless or not.

Although we can not observe β at the population level, we can compute its estimator as well as associated variance in the first pass of the FM two-pass procedure, which is sufficient for us to conduct a rank test, e.g. the rank test of Kleibergen and Paap (2006). There exist several rank tests, see Anderson (1951), Cragg and Donald (1996), etc. The rank test of Kleibergen and Paap (2006) is used in this paper as this novel test overcomes some deficiencies of other tests: it is robust to heteroscedasticity, while homoscedasticity is assumed in Anderson (1951); in addition, it is easier for implementation, while the rank test of Cragg and Donald (1996), which is used in Burnside (2010), involves numerical optimization.

Let $\hat{\beta}$ denote the estimator of β in the first pass time series regression of the FM two-pass procedure:

$$\hat{\beta} = RM_{\iota_T}F'(FM_{\iota_T}F')^{-1} \tag{8}$$

where $R = (R_1, R_2, ..., R_T)$, $F = (F_1, F_2, ..., F_T)$, $M_{\iota_T} = I_T - \iota_T(\iota_T'\iota_T)^{-1}\iota_T'$, I_T is the $T \times T$ identity matrix, ι_T is the $T \times 1$ vector of ones.

With $\hat{\beta}$ and its estimated variance, the rank test statistic $rk(q)$ of Kleibergen and Paap (2006) for the null hypothesis that H_0: rank$(\beta) = q$ can be computed, which asymptotically follows the χ^2 distribution with $(N-q)(m-q)$ degrees of freedom[10], $q < m < N$:

$$rk(q) \xrightarrow{d} \chi^2_{(N-q)(m-q)} \tag{9}$$

For our purpose, if non-traded factors are indeed useless or nearly useless, then we expect to see that the rank test suggests β has reduced rank; in contrast, if traded factors are more closely related to asset returns, then we expect to see that the rank test suggests β has full rank. For better illustration, we report the p value associated with the rank test statistic $rk(q)$ to describe the outcome of this test, and a p value lower than a preset level (e.g. 5%) implies the rejection of the null hypothesis that the rank of β is q.

3.1 Rank Test using 25 FF Portfolios

In Table 7, we use the commonly used 25 Fama-French size and book-to-market sorted portfolios as the test set to illustrate the rank test of Kleibergen and Paap (2006). In particular, we

[10] In details: $rk(q) = T\hat{\lambda}_q'\hat{\Omega}_q^{-1}\hat{\lambda}_q$, where $\hat{\lambda}_q = (\hat{B}_{q,\perp} \otimes \hat{A}'_{q,\perp})vec(\hat{\Theta})$, $\hat{\Theta} = G_1\hat{\beta}G_2$, and G_1, G_2 result from $(FF')^{-1} \otimes (RR') = (G_2'G_2)^{-1} \otimes (G_1'G_1)^{-1}$, $\hat{B}_{q,\perp}$, $\hat{A}'_{q,\perp}$ result from the singular value decomposition $\hat{\Theta} = \hat{A}_q\hat{B}_q + \hat{A}_{q,\perp}\hat{\lambda}_q\hat{B}_{q,\perp}$; $\hat{\Omega}_q = (\hat{B}_{q,\perp} \otimes \hat{A}'_{q,\perp})\hat{W}(\hat{B}_{q,\perp} \otimes \hat{A}'_{q,\perp})'$, where $\hat{W} = (G_2 \otimes G_1)\hat{V}(\hat{\beta})(G_2 \otimes G_1)'$, $\hat{V}(\hat{\beta})$ is the estimated variance of $\hat{\beta}$.

focus on the difference in test outcome (measured by p values) between non-traded and traded macroeconomic factors that are also studied in Section 2.

Take the Fama and French (1993) three factor model in Table 7 for example: R_M, SMB and HML are the three well-known factors. If these three factors are closely related to asset returns, then the β matrix in this model would have full rank, i.e. the rank equals 3. Although the true β is unknown, we can get its estimator $\hat{\beta}$ as well as the variance of this estimator in the first pass of the FM two-pass procedure. With the estimator and its variance, we apply the Kleibergen and Paap (2006) rank test to examine whether β has full rank by computing the rank test statistics, whose associated p values are reported in the right panel of Table 7. The rank test tests three null hypotheses that the rank of β is 0, 1 and 2 respectively (i.e. $H_0 : q = 0$, $H_0 : q = 1$, $H_0 : q = 2$), and reports three p values which are all approximately equal to 0.00 for these three null hypotheses. These small p values imply that we can reject all the three null hypotheses at 5% (1% as well) significance level, which further indicates that β has full rank 3, hence all the Fama-French factors are closely related to asset returns.

Similarly, let's now look at the durable consumption model of Yogo (2006), where R_M, $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ are the three risk factors. Table 7 shows that we can reject the null that the rank of β equals 0 at 5%; however, we can not reject the two hypotheses that the rank equals 1 or 2 at 5% when non-traded $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ are used, because of the high p values (0.37 and 0.69) associated with the two hypotheses. Consequently, the rank test indicates that only one factor among R_M, $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ is closely related to asset returns, while the other two factors are not. Not surprisingly, the conclusion based on the rank test is consistent with our empirical findings in Section 2, where we report that the non-traded $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ are only weakly related to the latent factors for asset returns. In contrast, if we use the traded version of $\triangle C_{Dur}$ and $\triangle C_{Ndur}$, and conduct the same rank test again, we find that all p values are now approximately equal to 0.00. These small p values indicate that the β matrix is likely to have full rank, which further implies that the traded $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ are more closely related to asset returns, compared to their non-traded counterparts.

Table 7 also reports the rank test outcome for the specifications in Cochrane (1996), Li et al. (2006) and Muir et al. (2011), hence all the non-traded and traded macroeconomic factors discussed in Section 2 are revisited in Table 7. Overall, the rank test results are consistent with our previous findings in Section 2: if the specifications of asset pricing models contain the non-traded risk factors that are not statistically related to the latent factors, then the rank test of Kleibergen and Paap (2006) applied at the first pass of the FM two-pass procedure exhibits large p values for reduced rank hypotheses, indicating that the correlation of the proposed risk factors and asset returns is weak; in contrast, when traded factors are used to replace non-traded factors, all p values have been greatly reduced, and most of them now lie below 5% (except for $p = 0.24$ associated with $rank(\beta) = 2$ in the Li et al. (2006) model). The reduction in p values indicates that the correlation of asset returns and risk factors gets stronger when traded factors are used to replace their non-traded counterparts. This is consistent with our previous findings in Section 2, where we report that traded factors appear to be more closely related to latent factors, the linear combination of which is the major component of asset returns in the linear factor model.

3.2 Rank Test using 25 FF + 30 Industry Portfolios

In Table 8, we augment the 25 Fama-French size and book-to-market sorted portfolios with the 30 industry portfolios from Kenneth French's web site, and similarly present the outcome of the Kleibergen and Paap (2006) rank test. This set of 25 plus 30 portfolios is proposed in Lewellen et al. (2010) to replace the conventional 25 size and book-to-market sorted portfolios as the test set, and the purpose is similarly to check whether our findings that traded factors are more closely related to asset returns remain unchanged when a different test set is used.

The p values in Table 8 are found to be similar to those in Table 7: firstly, the three Fama-French factors, R_M, SMB and HML, have p values approximately equal to zero, indicating that the corresponding β matrix has full rank; secondly, all p values become smaller, and most of them now lie below 5% (except for $p = 0.30$ associated with $rank(\beta) = 2$ in the Li et al. (2006) model), if we replace non-traded macroeconomic factors with their traded counterparts. In other words, p values of the Kleibergen and Paap (2006) rank test in Table 8 suggest that like the three Fama-French factors, traded macroeconomic factors are more closely related to asset returns, compared to non-traded macroeconomic factors. These findings thus do not contradict those in Table 7[11].

To summarize, Table 7 and 8 indicate that a rank test can be of help for detecting possibly

[11] We similarly used the 100 size and book-to-market sorted portfolios augmented by the 49 industry portfolios as the test set, and found qualitatively similar results of the rank test for β, which are omitted here for brevity.

weak or completely useless risk factors. When non-traded macroeconomic factors are used in asset pricing models, high p values of the Kleibergen and Paap (2006) rank test are commonly found in Table 7 and 8, which suggest that the full rank condition of the β matrix is probably at risk. These findings are consistent with the results in Section 2.

4 Risk Premium

So far, we have shown that many non-traded macroeconomic factors are unlikely to be the ideal proxies for the latent risk factors by presenting two pieces of evidence (the two sides of the same coin), which cast doubt on their seeming success in the FM two-pass procedure. The first evidence is that these proposed factors are not statistically related to the latent ones when we apply the methodology of Bai and Ng (2006); the second evidence is that these proposed factors are not statistically related to asset returns in the first pass of the FM two-pass procedure when we apply Kleibergen and Paap (2006)'s rank test. In contrast, there is some evidence that traded factors appear more closely related to latent factors as well as asset returns when we apply the approach of Bai and Ng (2006) or the rank test of Kleibergen and Paap (2006).

Recently, whether the proposed factors are closely related to asset returns has been shown to be crucial for the success of the FM two-pass procedure in finite sample applications. For example, Kleibergen (2009) proves that risk premium estimation in the second pass of the FM two-pass procedure is unreliable under useless or nearly useless risk factors, and Kleibergen and Zhan (2013) further show that when the proposed factors are completely or nearly useless, the second pass cross-sectional OLS R^2 is still likely to be large in empirical applications. Thus neither the large value of the cross-sectional R^2 nor the risk premium in the second pass of the FM two-pass procedure can be used as the evidence to support the proposed factors, without examining whether the proposed factors are closely related to asset returns in the first pass. Details of these results can be found in Kleibergen (2009) and Kleibergen and Zhan (2013) (e.g. Theorem 1 in Kleibergen (2009), and Theorem 3 in Kleibergen and Zhan (2013)).

In this section, we use the factor statistics proposed in Kleibergen (2009) to construct the C.I.'s of risk premium for both non-traded and traded macroeconomic factors that are also discussed in Section 2 and 3. The purpose is to explore whether the choice of non-traded or traded factors affect the inference on risk premium, which indicates whether the proposed factors are well priced. We use the factor statistics of Kleibergen (2009), because these factor statistics can produce trustworthy C.I.'s of risk premium no matter whether the proposed factors are strongly or weakly correlated with asset returns, while the t-statistic in the second-pass of the FM two-pass procedure is unreliable under weak or useless factors, which has been shown in Kleibergen (2009).

Specifically, Kleibergen (2009) advocates the usage of four identification robust factor statistics to replace the unreliable FM t-statistic, which can be inverted to derive the C.I.'s of risk premium: the factor Anderson-Rubin (FAR) statistic, the factor extension of Moreira (2003)'s conditional likelihood ratio statistic (FCLR), the factor extension of Kleibergen (2005)'s J-statistic (FJKLM) and the factor extension of Kleibergen's (2002, 2005) Lagrange multiplier statistic (FKLM). The 95% confidence intervals of risk premium constructed by inverting these test statistics are trustworthy no matter whether factors are useless or useful: if factors are useless, then confidence intervals of risk premium associated with them are unbounded, which reflect that these factors do not contain much information about risk premium; in contrast, if factors are useful, their associated confidence intervals of risk premium tend to be bounded.

In our application, we adopt the FCLR and FKLM statistics in Kleibergen (2009) as well as the FM t-statistic to construct the 95% C.I.'s of risk premium. We choose these two factor statistics in Kleibergen (2009) because they just test if risk premium is equal to a specific value while FAR and FJKLM also or just test if the mean return of assets is linearly spanned by β with some specific risk premium. The FM t-statistic is used here to provide a benchmark. We omit the mathematical expressions of these test statistics, which can be found in Kleibergen (2009) and Kleibergen and Zhan (2013).

Similar to Section 3, we first use the 25 Fama-French size and book-to-market sorted portfolios as the test set, then augment these portfolios with the 30 industry portfolios to construct the second test set, following the suggestion of Lewellen et al. (2010). The purpose is to see how the choice of test portfolios as well as the choice of non-traded and traded factors, affects the C.I.'s of risk premium.

4.1 Risk Premium using 25 FF Portfolios

In Figure 1-5, we present the C.I.'s of risk premium constructed by the two identification robust factor statistics of Kleibergen (2009) (namely FCLR and FKLM) and the conventional FM t-statistic. Each figure corresponds to one of the five asset pricing models studied in Table 7 and 8 respectively, and the same data as used for Table 7 is used to draw these figures. In particular, we compare the risk premium associated with non-traded and traded factors in Figure 2-5. We expect to see unbounded C.I.'s associated with non-traded factors, and bounded C.I.'s associated with traded factors, given we have found that traded factors appear stronger than their non-traded counterparts in terms of the correlation with asset returns, hence are more likely to be priced.

Figure 1 contains the one minus p value plots for the risk premium on the three Fama-French factors, and each plot corresponds to one of the three statistics, namely FCLR, FKLM and FM t-statistic. The 95% C.I. of risk premium is the interval bounded by the two points at which each p value plot of the three statistics intersects the straight 0.95 line. For example, by inverting the FM t-statistic (see the solid line in Figure 1), the 95% C.I. of risk premium associated with R_M is approximately $(-4.6, -0.1)$, and these two values are approximately equal to the point estimate of risk premium plus/minus 1.96 times standard error reported in Table 7. The 95% C.I.'s constructed by inverting the two factor statistics FCLR and FKLM can be similarly read from the figure. Figure 1 shows that all three Fama-French factors have bounded C.I.'s of risk premium, hence they are well priced. In addition, C.I.'s by FM t-statistic are comparable to C.I.'s by factor statistics of Kleibergen (2009), which further indicates that the three Fama-French factors are good proxies for the latent factors.

Figure 2 shows the one minus p value plots for the risk premium associated with the two factors in Cochrane (1996), $\triangle I_{Nres}$ and $\triangle I_{Res}$. On the left column, we present the outcome associated with the two non-traded factors, while on the right column, we show the outcome associated with the two corresponding traded factors. The p value plots on the left show that the confidence intervals for non-traded $\triangle I_{Nres}$ and $\triangle I_{Res}$ by two factor statistics are unbounded since these plots do not cross the 0.95 line twice; in contrast, the p value plots on the right show that the confidence intervals of risk premium for traded $\triangle I_{Nres}$ and $\triangle I_{Res}$ are bounded. As a result, Figure 2 conveys the message that traded factors appear more informative than their non-traded counterparts. Again, the findings in Figure 2 are consistent with those in Section 2 and 3, where we report that the traded $\triangle I_{Nres}$ and $\triangle I_{Res}$ are more closely related to latent factors and asset returns, compared to their non-traded counterparts.

Figure 3-5 can be similarly interpreted as Figure 2, although the pattern in Figure 3-5 is not as obvious as in Figure 2. Similar to Figure 2, the left columns of Figure 3-5 show that C.I.'s for non-traded factors by factor statistics are all unbounded and substantially different from C.I.'s by the FM t-statistic, which suggests that non-traded factors are neither informative for risk premium nor good proxies for latent factors. When we replace non-traded factors with their traded counterparts, the confidence intervals are presented on the right columns of Figure 3-5. Unlike the right column of Figure 2, the right columns of Figure 3-5 do not clearly show that confidence intervals by factor statistics are bounded, although there seems to be the tendency towards that.

To summarize, Figure 1-5 indicate that risk premium implied by many non-traded macroeconomic factors are not very informative. However, there is some evidence that traded factors are more appropriate than non-traded factors in the first pass time series regression of the FM two-pass procedure, since they are more likely to generate informative confidence intervals of risk premium.

4.2 Risk Premium using 25 FF + 30 Industry Portfolios

Instead of the 25 size and book-to-market sorted portfolios, we now use these 25 portfolios augmented by the 30 industry portfolios from Kenneth French's web site, and draw one minus p value plots in Figure 6-10, which are the mirror images of Figure 1-5 but produced with different data. Each figure of Figure 6-10 similarly corresponds to one of the five asset pricing models studied in Table 7 and 8, and the same data for Table 8 is used to draw these figures. As in Figure 2-5, the left columns of Figure 7-10 present confidence intervals of risk premium for non-traded macroeconomic factors, while the right columns are for the traded factors.

Similar to Figure 1, Figure 6 contains the one minus p value plots for risk premium on the three Fama-French factors. Again, we find that the implied 95% C.I.'s of risk premium associated with all the three Fama-French factors are bounded; in addition, C.I.'s by factor statistics of Kleibergen (2009) are not too far from C.I.'s by FM t-statistic, which suggests that the three Fama-French factors are the good proxies for latent factors.

Similar to Figure 2, Figure 7 also indicates that traded $\triangle I_{Nres}$ and $\triangle I_{Res}$ in the Cochrane (1996) model are more informative for risk premium, compared to their non-traded counterparts, whose C.I.'s of risk premium on the left column of Figure 7 are unbounded.

Compared to Figure 3, it is now more obvious in Figure 8 that using traded $\triangle C_{Dur}$ and $\triangle C_{Ndur}$ improves the inference on risk premium in the Yogo (2006) model. For example, both FCLR and FKLM imply bounded C.I.'s of risk premium associated with the traded $\triangle C_{Ndur}$; in addition, FCLR also implies the bounded C.I. of risk premium associated with the traded $\triangle C_{Dur}$, while C.I.'s associated with non-traded $\triangle C_{Dur}$ are unbounded.

Figure 9 is a bit uneasy to interpret. For non-traded factors on the left column, FCLR and FKLM yield bounded C.I.'s, while C.I.'s for traded factors become unbounded on the right column. Hence it appears that traded factors in the Li et al. (2006) model are less informative for risk premium, compared to non-traded factors. However, the bounded C.I.'s by FCLR and FKLM on the left column of Figure 9 are much wider than C.I.'s by FM t-statistic, and the C.I.'s for $Finan$ also substantially differ. Hence the validity of the three non-traded factors is still under doubt. Furthermore, since the traded factors in the Li et al. (2006) model also yield some unusual results in Section 2 and 3 (i.e. low $R^2(j)$ and still sizeable p values when traded factors are used, see Table 6-8), it is not surprising that Figure 9 shows traded factors in this model appear uninformative.

Figure 10 is in line with Figure 7 and 8, i.e. the traded Lev factor appears more informative for risk premium than its non-traded version, since C.I.'s by FCLR and FKLM are unbounded on the left, but bounded on the right column of Figure 10. However, the substantial difference between C.I.'s by FCLR, FKLM and C.I. by FM t-statistic also casts doubt on the Lev factor proposed in Muir et al. (2011).

Based on Figure 1-10 briefly described above, it appears that many non-traded macroeconomic factors are very uninformative for risk premium; furthermore, in most cases, using traded factors instead of non-traded ones can help improve the information we want for risk premium, although the improvement may be rather minor (see Figure 2, 3, 4, 5, 7, 8, 10, left column vs. right column)

Finally, it is not unusual in Figure 1-10 that the factor statistics in Kleibergen (2009) and the conventional FM t-statistic often produce quite different confidence intervals of risk premium.

Given that the performance of FM t-statistic crucially depends on the statistical quality of the proposed macroeconomic factors (many of which are not closely related to latent factors and asset returns as shown in Section 2-3), while the factor statistics in Kleibergen (2009) remain trustworthy under useless or nearly useless factors, it is expected that confidence intervals by FM t-statistic may substantially differ from those by the factor statistics. These substantial differences thus also serve as evidence for the unsatisfactory quality of the macroeconomic factors that have been discussed throughout Section 2, 3 and 4.

5 Conclusion

In the asset pricing literature, both non-traded macroeconomic factors and their traded counterparts are commonly used in the popular FM two-pass procedure. In this paper, we argue that it is advisable to use traded factors rather than non-traded ones from the econometric perspective, because we find the compelling evidence that non-traded factors are weakly correlated with asset returns, while Kleibergen (2009), Kleibergen and Zhan (2013) have demonstrated that weak correlation between the proposed factors and asset returns implies spurious statistical findings in the FM two-pass procedure.

To illustrate the weak correlation between non-traded macroeconomic factors and asset returns as well as the implied inference problem on risk premium, we adopt three methods.

The first method from Bai and Ng (2006) involves the estimation of the latent factors for asset returns by principal component analysis and the regression of the proposed non-traded or traded factors on the estimated latent factors. In the empirical application, we find that many of the non-traded macroeconomic factors do not seem to be strongly related to the latent factors for asset returns, while their traded counterparts appear to perform better. Our findings are consistent with Lewellen et al. (2010) in the sense that the various risk factors proposed in the asset pricing literature may not be the reasonable proxies for the systematic risk that could drive financial asset returns, however, our explanation differs from Lewellen et al. (2010) in that we emphasize the weak correlation between these proposed factors and the latent factors, and this weak correlation may induce spurious results in the FM two-pass procedure that seem to favor the proposed factors.

Secondly, by applying the rank test of Kleibergen and Paap (2006) to β, the correlation matrix of asset returns and proposed factors, we can not reject the null hypotheses that the β matrix has reduced rank under non-traded macroeconomic factors; in contrast, β is more likely to have full rank when traded factors are used. The outcome of the rank test thus suggests that non-traded factors are not as closely related to asset returns as traded factors, which is consistent with our findings based on the Bai and Ng (2006) approach, where traded factors are found to be more closely related to latent factors and thus asset returns.

Thirdly, given the above weak correlation between non-traded factors and latent factors as well as asset returns, we construct confidence intervals of risk premium by inverting the factor statistics in Kleibergen (2009), as these intervals are known to remain trustworthy no matter when factors are strongly or weakly correlated with asset returns. We report that non-traded macroeconomic factors tend to have unbounded confidence intervals, and hence contain little information for risk premium, while the inference for risk premium using traded factors appears more informative.

It is worth emphasizing that this paper is not meant to reject the non-traded macroeconomic factors in the asset pricing literature. Although empirical support for non-traded macroeconomic factors often comes from the FM two-pass procedure, these factors are typically proposed based on some theoretical model. This paper does not discuss any theoretical model where non-traded macroeconomic factors result from, but stresses that many of these factors appear to be weakly related to the latent factors and asset returns in a linear factor model, which raises the concern that the empirical evidence to support these factors based on the FM two-pass procedure is under doubt, following the work of Kan and Zhang (1999), Kleibergen (2009), Lewellen et al. (2010) and Kleibergen and Zhan (2013). In addition, we also report that the information for risk premium derived from the non-traded macroeconomic factors is rather limited.

We conclude by making three practical suggestions. Firstly, compared to non-traded factors, traded factors appear to be more closely related to asset returns, hence it is probably safer to use the traded version of the macroeconomic factors in order to avoid the failure in the FM two-pass procedure due to the poor statistical quality of factors. Secondly, it is advisable to use either the regression approach of Bai and Ng (2006) or the rank test of Kleibergen and Paap (2006) to examine the correlation of risk factors and asset returns in the first pass before conducting the FM two-pass procedure, which can make the second pass results more credible. Thirdly, if risk premium is of interest, it is helpful to use the factor statistics proposed by Kleibergen (2009) to accompany the FM t-statistic, as the difference in confidence intervals of risk premium implied by these statistics helps signal the quality of the proposed factors.

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Table 1: Summary Statistics

	Non-traded factors				
	obs	mean	s.e.	min	max
$\triangle C_{Dur}$	235	.0093	.0055	-.0049	.0203
$\triangle C_{Ndur}$	235	.0046	.0052	-.0125	.0236
$\triangle I_{Res}$	235	.0136	.0476	-.1658	.1760
$\triangle I_{Nres}$	235	.0165	.0271	-.0935	.1026
$Hholds$	235	.0142	.0412	-.1497	.1450
$Finan$	235	.0232	.0398	-.1153	.2538
$Nfinco$	235	.0192	.0815	-.3139	.2306
Lev	168	2.1735	13.5457	-59.9883	35.9422
	Traded factors				
	obs	mean	s.e.	min	max
$\triangle C_{Dur}$	600	.0907	3.6906	-13.8601	30.2400
$\triangle C_{Ndur}$	600	.1562	3.7695	-13.3687	29.7645
$\triangle I_{Res}$	600	.0509	3.5900	-14.1607	33.6023
$\triangle I_{Nres}$	600	.2635	3.0501	-14.8284	30.2192
$Hholds$	600	.0987	3.4680	-11.8608	32.7836
$Finan$	600	.2112	2.9153	-19.4597	27.9124
$Nfinco$	600	.0807	2.8747	-16.8046	27.2490
Lev	444	.2935	3.7635	-30.6051	26.4647
	Fama-French factors				
	obs	mean	s.e.	min	max
R_M	600	.4574	4.5254	-23.1400	16.0500
SMB	600	.2430	3.1283	-16.3900	22.0000
HML	600	.4221	2.8915	-12.6000	13.8400

Note: The non-traded macroeconomic factors are between 1952Q2 and 2010Q4, while the traded factors including the three Fama-French factors are between 1961M1 and 2010M12. We have limited data for the funding liquidity factor denoted by Lev: for its non-traded version, we have data between 1968Q1 and 2009Q4; for its traded version, we have data between 1973M1 and 2009M12.

Table 2: Correlation

Non-traded factors								
	$\triangle C_{Dur}$	$\triangle C_{Ndur}$	$\triangle I_{Res}$	$\triangle I_{Nres}$	$Hholds$	$Finan$	$Nfinco$	Lev
$\triangle C_{Dur}$	1.0000							
$\triangle C_{Ndur}$	0.3043	1.0000						
$\triangle I_{Res}$	0.2701	0.4085	1.0000					
$\triangle I_{Nres}$	0.2790	0.1562	0.2636	1.0000				
$Hholds$	0.2520	0.4003	0.9873	0.2706	1.0000			
$Finan$	0.2149	0.0602	0.1556	0.6405	0.1628	1.0000		
$Nfinco$	0.1226	0.0876	0.3086	0.4494	0.2990	0.2826	1.0000	
Lev	-0.0396	0.0993	0.0353	0.0869	0.0316	0.0418	-0.0501	1.0000

Traded factors								
	$\triangle C_{Dur}$	$\triangle C_{Ndur}$	$\triangle I_{Res}$	$\triangle I_{Nres}$	$Hholds$	$Finan$	$Nfinco$	Lev
$\triangle C_{Dur}$	1.0000							
$\triangle C_{Ndur}$	0.1817	1.0000						
$\triangle I_{Res}$	0.2548	0.7045	1.0000					
$\triangle I_{Nres}$	0.4641	0.4639	0.3717	1.0000				
$Hholds$	0.1835	0.6983	0.9711	0.3506	1.0000			
$Finan$	0.2862	0.2485	0.2013	0.6515	0.2878	1.0000		
$Nfinco$	0.4355	0.1958	0.1590	0.6762	0.1477	0.5255	1.0000	
Lev	-0.0681	-0.0704	0.0231	-0.2800	0.0800	-0.0734	-0.2044	1.0000

Note: The non-traded macroeconomic factors are between 1952Q2 and 2010Q4, while the traded factors are between 1961M1 and 2010M12. We have limited data for the funding liquidity factor denoted by Lev: for its non-traded version, we have data between 1968Q1 and 2009Q4; for its traded version, we have data between 1973M1 and 2009M12.

Table 3: Testing Non-traded Factors using 100 FF + 49 Industry Portfolios

($N = 131$, $T = 235$, $1952Q2 - 2010Q4$)

$k=5$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.945	110.233	0.019 (0.004, 0.054)	52.552	0.019 (0.004, 0.054)
	$\triangle C_{Dur}$	0.945	63.108	0.029 (0.000, 0.072)	33.071	0.220 (0.126, 0.313)
	$\triangle C_{Ndur}$	0.838	51.979	0.126 (0.047, 0.206)	6.916	0.060 (0.001, 0.119)
	$\triangle I_{Res}$	0.881	70.200	0.087 (0.018, 0.156)	10.476	0.026 (0.000, 0.066)
	$\triangle I_{Nres}$	0.915	107.697	0.037 (0.000, 0.085)	25.812	0.011 (0.000, 0.037)
	$Hholds$	0.864	75.284	0.078 (0.012, 0.144)	11.821	0.002 (0.000, 0.012)
	$Finan$	0.932	376.036	0.011 (0.000, 0.038)	86.775	0.000 (0.000, 0.000)
	$Nfinco$	0.889	73.231	0.029 (0.000, 0.072)	32.989	0.000 (0.000, 0.000)
	Lev	0.851	80.956	0.104 (0.016, 0.191)	8.650	0.104 (0.016, 0.191)
$k=6$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.932	89.601	0.022 (0.006, 0.059)	44.392	0.022 (0.006, 0.059)
	$\triangle C_{Dur}$	0.940	58.192	0.031 (0.000, 0.074)	31.717	0.224 (0.130, 0.318)
	$\triangle C_{Ndur}$	0.847	48.703	0.128 (0.049, 0.208)	6.785	0.061 (0.002, 0.120)
	$\triangle I_{Res}$	0.855	77.172	0.089 (0.020, 0.159)	10.209	0.029 (0.000, 0.072)
	$\triangle I_{Nres}$	0.902	92.864	0.040 (0.000, 0.088)	24.282	0.025 (0.000, 0.064)
	$Hholds$	0.894	83.599	0.079 (0.013, 0.145)	11.639	0.008 (0.000, 0.031)
	$Finan$	0.923	288.776	0.013 (0.000, 0.042)	75.217	0.000 (0.000, 0.005)
	$Nfinco$	0.889	68.603	0.032 (0.000, 0.076)	30.681	0.000 (0.000, 0.000)
	Lev	0.869	79.054	0.104 (0.017, 0.191)	8.611	0.104 (0.017, 0.191)

Note: $A(j)$ is the frequency that $|\hat{\tau}_t(j)|$ exceeds the 5% asymptotic critical value; $M(j)$, $R^2(j)$, $NS(j)$, $\hat{\rho}(k)^2$ are the statistics defined in Section 2. Specifically, $R^2(j)$ stands for the R-squared when we regress each listed risk factor on the k latent factors. 95% C.I.'s are in the brackets. N(0,1) stands for a useless factor simulated from the standard normal distribution. $\hat{\rho}(k)^2$'s for N(0,1) and Lev are equal to their R-squared since they are computed separately, while the other $\hat{\rho}(k)^2$'s are the ordered canonical correlations between the risk factors ($\triangle C_{Dur}$, $\triangle C_{Ndur}$, $\triangle I_{Res}$, $\triangle I_{Nres}$, $Hholds$, $Finan$, $Nfinco$) and the latent factors.

Table 4: Testing Non-traded Factors using 25 FF + 30 Industry Portfolios

($N = 55$, $T = 235$, $1952Q2 - 2010Q4$)

$k = 8$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.902	67.191	0.031 (0.009, 0.073)	30.896	0.031 (0.009, 0.073)
	$\triangle C_{Dur}$	0.923	65.655	0.039 (0.000, 0.087)	24.766	0.236 (0.141, 0.331)
	$\triangle C_{Ndur}$	0.847	53.296	0.128 (0.048, 0.208)	6.804	0.086 (0.017, 0.154)
	$\triangle I_{Res}$	0.851	42.046	0.108 (0.033, 0.183)	8.227	0.050 (0.000, 0.104)
	$\triangle I_{Nres}$	0.881	70.227	0.063 (0.003, 0.124)	14.751	0.042 (0.000, 0.093)
	$Hholds$	0.843	43.909	0.100 (0.027, 0.173)	9.016	0.010 (0.000, 0.036)
	$Finan$	0.864	151.053	0.031 (0.000, 0.074)	31.554	0.007 (0.000, 0.028)
	$Nfinco$	0.826	51.338	0.061 (0.002, 0.120)	15.430	0.000 (0.000, 0.005)
	Lev	0.804	75.198	0.109 (0.020, 0.198)	8.178	0.109 (0.020, 0.198)
$k = 9$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.894	62.648	0.036 (0.012, 0.078)	26.930	0.036 (0.012, 0.078)
	$\triangle C_{Dur}$	0.940	53.386	0.042 (0.000, 0.092)	22.980	0.262 (0.166, 0.359)
	$\triangle C_{Ndur}$	0.847	53.446	0.130 (0.049, 0.210)	6.719	0.094 (0.023, 0.165)
	$\triangle I_{Res}$	0.834	46.595	0.110 (0.035, 0.186)	8.052	0.064 (0.003, 0.124)
	$\triangle I_{Nres}$	0.898	59.368	0.069 (0.006, 0.131)	13.593	0.043 (0.000, 0.093)
	$Hholds$	0.851	49.994	0.102 (0.029, 0.175)	8.795	0.027 (0.000, 0.068)
	$Finan$	0.847	104.393	0.038 (0.000, 0.087)	25.019	0.010 (0.000, 0.036)
	$Nfinco$	0.787	37.719	0.089 (0.019, 0.158)	10.258	0.000 (0.000, 0.006)
	Lev	0.768	51.702	0.119 (0.027, 0.211)	7.382	0.119 (0.027, 0.211)

Note: $A(j)$ is the frequency that $|\hat{\tau}_t(j)|$ exceeds the 5% asymptotic critical value; $M(j)$, $R^2(j)$, $NS(j)$, $\hat{\rho}(k)^2$ are the statistics defined in Section 2. Specifically, $R^2(j)$ stands for the R-squared when we regress each listed risk factor on the k latent factors. 95% C.I.'s are in the brackets. N(0,1) stands for a useless factor simulated from the standard normal distribution. $\hat{\rho}(k)^2$'s for N(0,1) and Lev are equal to their R-squared since they are computed separately, while the other $\hat{\rho}(k)^2$'s are the ordered canonical correlations between the risk factors ($\triangle C_{Dur}$, $\triangle C_{Ndur}$, $\triangle I_{Res}$, $\triangle I_{Nres}$, $Hholds$, $Finan$, $Nfinco$) and the latent factors.

Table 5: Testing Non-traded Factors using CRSP Stocks

($N = 1411$, $T = 80$, $1991Q1 - 2010Q4$)

$k = 3$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.975	211.862	0.028 (0.003, 0.112)	34.281	0.028 (0.003, 0.112)
	$\triangle C_{Dur}$	1.000	173.789	0.014 (0.000, 0.065)	70.844	0.127 (0.000, 0.263)
	$\triangle C_{Ndur}$	0.988	206.838	0.070 (0.000, 0.178)	13.239	0.057 (0.000, 0.156)
	$\triangle I_{Res}$	1.000	180.111	0.036 (0.000, 0.115)	27.093	0.020 (0.000, 0.080)
	$\triangle I_{Nres}$	0.975	159.021	0.021 (0.000, 0.084)	45.571	0.000 (0.000, 0.000)
	$Hholds$	0.988	167.476	0.033 (0.000, 0.109)	29.742	0.000 (0.000, 0.000)
	$Finan$	0.975	399.733	0.033 (0.000, 0.110)	29.360	0.000 (0.000, 0.000)
	$Nfinco$	0.988	363.945	0.011 (0.000, 0.056)	92.333	0.000 (0.000, 0.000)
	Lev	0.934	152.565	0.077 (0.000, 0.193)	11.940	0.077 (0.000, 0.193)
$k = 4$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	N(0,1)	0.975	155.449	0.041 (0.006, 0.134)	22.237	0.041 (0.006, 0.134)
	$\triangle C_{Dur}$	0.988	116.084	0.028 (0.000, 0.100)	34.529	0.203 (0.046, 0.360)
	$\triangle C_{Ndur}$	0.963	112.610	0.124 (0.000, 0.259)	7.077	0.123 (0.000, 0.258)
	$\triangle I_{Res}$	0.988	142.322	0.045 (0.000, 0.133)	21.307	0.057 (0.000, 0.156)
	$\triangle I_{Nres}$	0.950	73.520	0.091 (0.000, 0.211)	9.996	0.013 (0.000, 0.063)
	$Hholds$	0.975	135.921	0.044 (0.000, 0.132)	21.797	0.000 (0.000, 0.000)
	$Finan$	0.975	174.420	0.057 (0.000, 0.156)	16.499	0.000 (0.000, 0.000)
	$Nfinco$	0.887	106.345	0.075 (0.000, 0.187)	12.258	0.000 (0.000, 0.000)
	Lev	0.934	141.036	0.082 (0.000, 0.201)	11.145	0.082 (0.000, 0.201)

Note: $A(j)$ is the frequency that $|\hat{\tau}_t(j)|$ exceeds the 5% asymptotic critical value; $M(j)$, $R^2(j)$, $NS(j)$, $\hat{\rho}(k)^2$ are the statistics defined in Section 2. Specifically, $R^2(j)$ stands for the R-squared when we regress each listed risk factor on the k latent factors. 95% C.I.'s are in the brackets. N(0,1) stands for a useless factor simulated from the standard normal distribution. $\hat{\rho}(k)^2$'s for N(0,1) and Lev are equal to their R-squared since they are computed separately, while the other $\hat{\rho}(k)^2$'s are the ordered canonical correlations between the risk factors ($\triangle C_{Dur}$, $\triangle C_{Ndur}$, $\triangle I_{Res}$, $\triangle I_{Nres}$, $Hholds$, $Finan$, $Nfinco$) and the latent factors.

Table 6: Testing Traded Factors using 100 FF + 49 Industry Portfolios

($N = 131$, $T = 600$, $1961M1 - 2010M12$)

$k=6$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	R_M	0.123	5.487	0.984 (0.982, 0.987)	0.016	0.992 (0.991, 0.994)
	SMB	0.078	4.010	0.959 (0.953, 0.966)	0.043	0.968 (0.963, 0.973)
	HML	0.180	5.861	0.906 (0.892, 0.920)	0.104	0.887 (0.870, 0.904)
	$\triangle C_{Dur}$	0.775	42.318	0.098 (0.053, 0.143)	9.205	0.367 (0.306, 0.428)
	$\triangle C_{Ndur}$	0.595	20.331	0.335 (0.273, 0.396)	1.987	0.188 (0.132, 0.244)
	$\triangle I_{Res}$	0.653	28.183	0.381 (0.320, 0.443)	1.622	0.127 (0.077, 0.177)
	$\triangle I_{Nres}$	0.645	30.034	0.146 (0.094, 0.199)	5.837	0.000 (0.000, 0.000)
	$Hholds$	0.652	30.219	0.355 (0.294, 0.417)	1.817	0.000 (0.000, 0.000)
	$Finan$	0.687	28.582	0.183 (0.127, 0.239)	4.475	0.000 (0.000, 0.000)
	$Nfinco$	0.833	69.455	0.044 (0.012, 0.076)	21.924	0.000 (0.000, 0.000)
	Lev	0.651	26.343	0.267 (0.197, 0.338)	2.740	0.267 (0.197, 0.338)
$k=7$	Factor j	$A(j)$	$M(j)$	$R^2(j)$	$NS(j)$	$\hat{\rho}(k)^2$
	R_M	0.087	4.969	0.987 (0.985, 0.989)	0.013	0.993 (0.992, 0.994)
	SMB	0.045	4.066	0.965 (0.959, 0.970)	0.037	0.969 (0.964, 0.974)
	HML	0.142	4.876	0.920 (0.907, 0.932)	0.087	0.910 (0.897, 0.924)
	$\triangle C_{Dur}$	0.733	37.493	0.123 (0.073, 0.172)	7.163	0.384 (0.323, 0.445)
	$\triangle C_{Ndur}$	0.595	22.375	0.340 (0.278, 0.401)	1.944	0.188 (0.132, 0.244)
	$\triangle I_{Res}$	0.647	26.724	0.387 (0.326, 0.448)	1.581	0.128 (0.078, 0.177)
	$\triangle I_{Nres}$	0.625	34.700	0.161 (0.107, 0.215)	5.216	0.055 (0.020, 0.091)
	$Hholds$	0.648	28.576	0.360 (0.299, 0.422)	1.776	0.000 (0.000, 0.000)
	$Finan$	0.668	34.578	0.188 (0.132, 0.244)	4.316	0.000 (0.000, 0.000)
	$Nfinco$	0.727	48.292	0.095 (0.051, 0.140)	9.495	0.000 (0.000, 0.000)
	Lev	0.667	25.385	0.274 (0.203, 0.344)	2.653	0.274 (0.203, 0.344)

Note: $A(j)$ is the frequency that $|\hat{\tau}_t(j)|$ exceeds the 5% asymptotic critical value; $M(j)$, $R^2(j)$, $NS(j)$, $\hat{\rho}(k)^2$ are the statistics defined in Section 2. Specifically, $R^2(j)$ stands for the R-squared when we regress each listed risk factor on the k latent factors. 95% C.I.'s are in the brackets. $\hat{\rho}(k)^2$ for Lev is equal to its R-squared since it is computed separately, while the other $\hat{\rho}(k)^2$'s are the ordered canonical correlations between the risk factors (R_M, SMB, HML, $\triangle C_{Dur}$, $\triangle C_{Ndur}$, $\triangle I_{Res}$, $\triangle I_{Nres}$, $Hholds$, $Finan$, $Nfinco$) and the latent factors.

Table 7: Rank Test for β in 5 Asset Pricing Models using 25 FF Portfolios

Model	Factors			R^2	p value of H_0 : rank=0	rank=1	rank=2
Fama-French (1993)	R_M	SMB	HML				
	-2.356	0.641	1.441	0.79	0.00	0.00	0.00
	(1.140)	(0.098)	(0.120)				
Cochrane (1996)	$\triangle I_{Nres}$	$\triangle I_{Res}$					
Non-traded	0.014	0.041		0.31	0.17	0.31	
	(0.016)	(0.021)					
Traded	-6.771	-1.296		0.49	0.00	0.00	
	(2.366)	(1.355)					
Yogo (2006)	R_M	$\triangle C_{Ndur}$	$\triangle C_{Dur}$				
Non-traded	1.748	0.714	0.019	0.37	0.00	0.37	0.69
	(2.595)	(0.409)	(0.297)				
Traded	-1.106	0.651	-5.193	0.78	0.00	0.00	0.00
	(1.395)	(1.197)	(2.082)				
Li-Vassalou-Xing (2006)	$Hholds$	$Nfinco$	$Finan$				
Non-traded	0.047	0.043	0.009	0.36	0.06	0.22	0.37
	(0.026)	(0.038)	(0.020)				
Traded	-0.532	-4.592	1.191	0.69	0.00	0.03	0.24
	(1.314)	(1.938)	(2.825)				
Muir et al. (2011)	Lev						
Non-traded	13.836			0.74	0.14		
	(6.057)						
Traded	6.301			0.70	0.00		
	(2.437)						

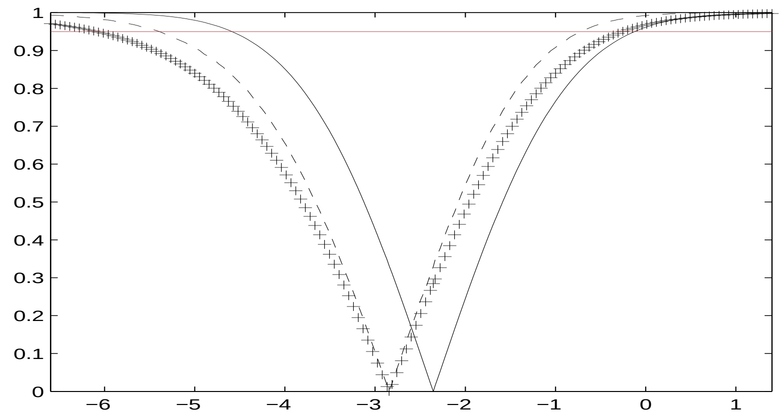
Note: The table presents the p values of the Kleibergen and Paap (2006) rank test for 5 asset pricing models, including the benchmark Fama-French three factor model. The null hypotheses that the β matrix has reduced rank are tested. For each model, the OLS estimates of the risk premium and the cross-sectional OLS R^2 are also reported, where standard errors with Shanken (1992) correction are in brackets. The quarterly returns of 25 Fama-French size and book-to-market sorted portfolios during 1973Q1-2009Q4 are used as the test set. All risk factors (three Fama-French factors plus non-traded and traded macroeconomic factors) that are examined in Table 1-6 are covered.

Table 8: Rank Test for β in 5 Asset Pricing Models using 25 FF + 30 Industry Portfolios

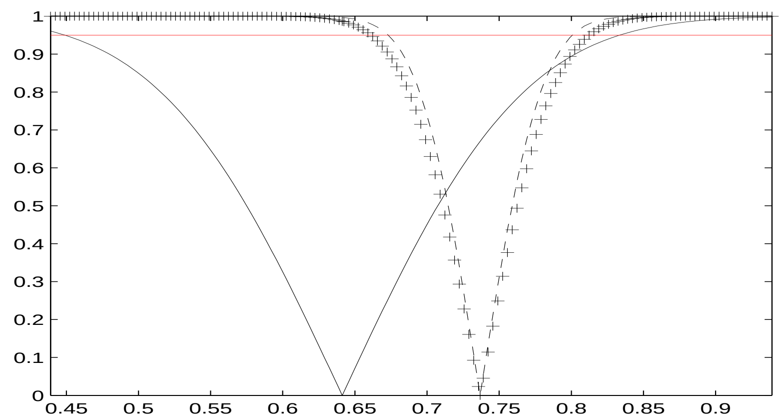
Model	Factors			R^2	p value of H_0 : rank=0	rank=1	rank=2
Fama-French (1993)	R_M	SMB	HML				
	-1.410	0.454	1.003	0.36	0.00	0.00	0.00
	(0.853)	(0.139)	(0.175)				
Cochrane (1996)	$\triangle I_{Nres}$	$\triangle I_{Res}$					
Non-traded	0.000	0.004		0.01	0.14	0.33	
	(0.005)	(0.010)					
Traded	-0.391	-0.042		0.01	0.00	0.00	
	(1.091)	(0.657)					
Yogo (2006)	R_M	$\triangle C_{Ndur}$	$\triangle C_{Dur}$				
Non-traded	-0.140	0.043	0.125	0.05	0.00	0.30	0.47
	(0.887)	(0.095)	(0.137)				
Traded	-0.535	0.261	-2.256	0.28	0.00	0.00	0.00
	(0.941)	(1.032)	(1.139)				
Li-Vassalou-Xing (2006)	$Hholds$	$Nfinco$	$Finan$				
Non-traded	0.009	0.009	-0.015	0.13	0.11	0.49	0.71
	(0.010)	(0.015)	(0.009)				
Traded	0.145	-1.217	1.146	0.21	0.00	0.01	0.30
	(0.806)	(1.078)	(1.302)				
Muir et al. (2011)	Lev						
Non-traded	5.260			0.16	0.55		
	(3.691)						
Traded	0.471			0.01	0.00		
	(0.926)						

Note: The table presents the p values of the Kleibergen and Paap (2006) rank test for 5 asset pricing models, including the benchmark Fama-French three factor model. The null hypotheses that the β matrix has reduced rank are tested. For each model, the OLS estimates of the risk premium and the cross-sectional OLS R^2 are also reported, where standard errors with Shanken (1992) correction are in brackets. The quarterly returns of 25 Fama-French size and book-to-market sorted portfolios and 30 industry portfolios during 1973Q1-2009Q4 are used as the test set. All risk factors (three Fama-French factors plus non-traded and traded macroeconomic factors) that are examined in Table 1-6 are covered.

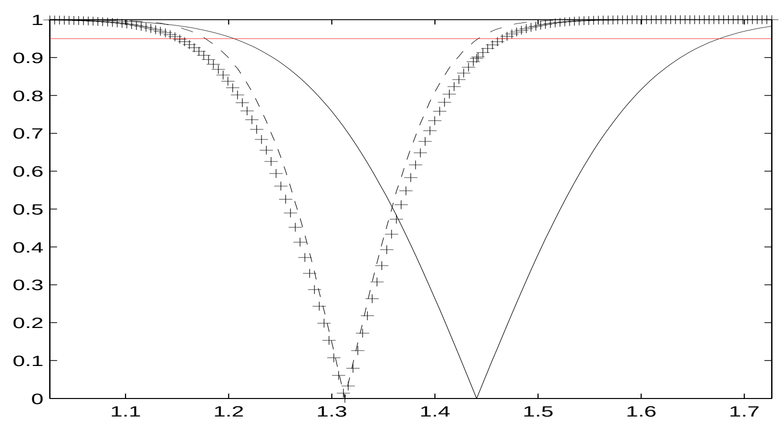
Figure 1: $1 - p$ plots for risk premium in Fama-French (1993) using 25 FF Portfolios



(a) R_M



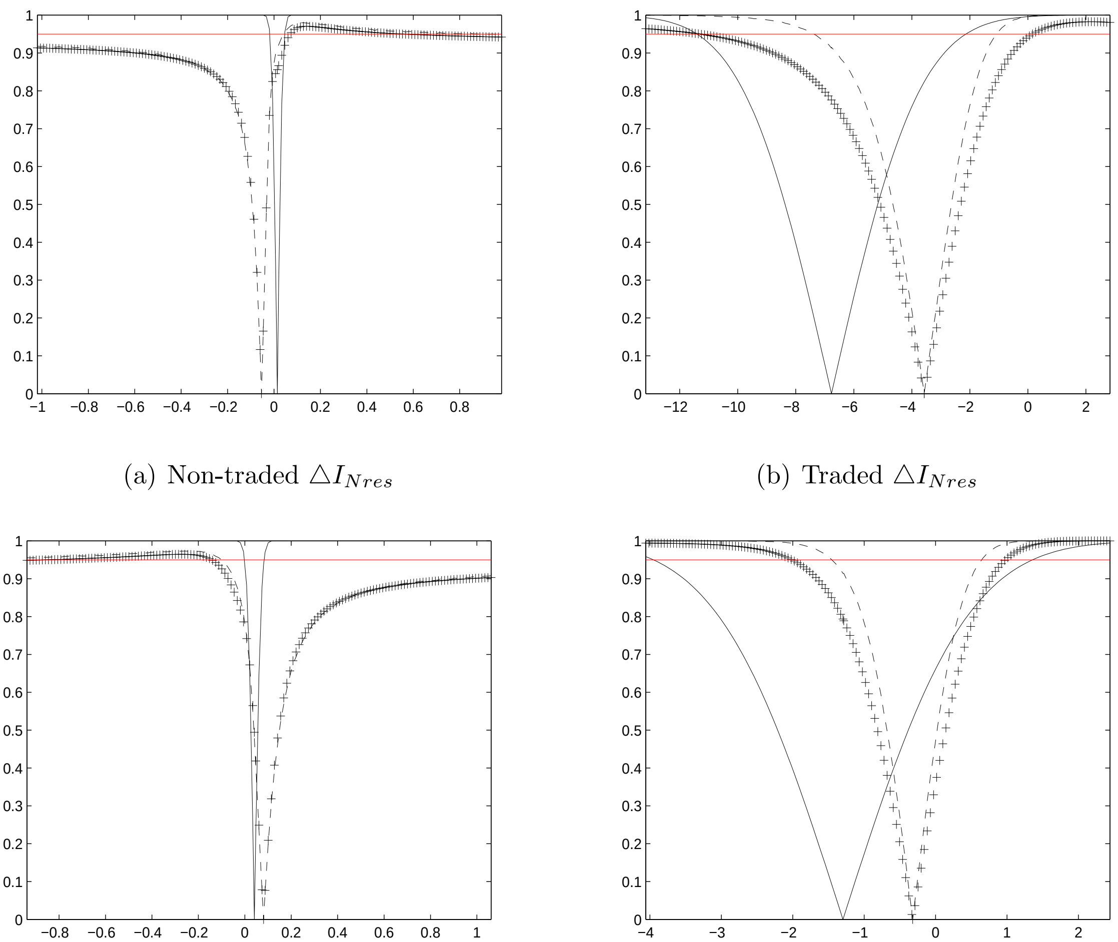
(b) SMB



(c) HML

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

Figure 2: $1 - p$ plots for risk premium in Cochrane (1996) using 25 FF Portfolios



(a) Non-traded $\triangle I_{Nres}$

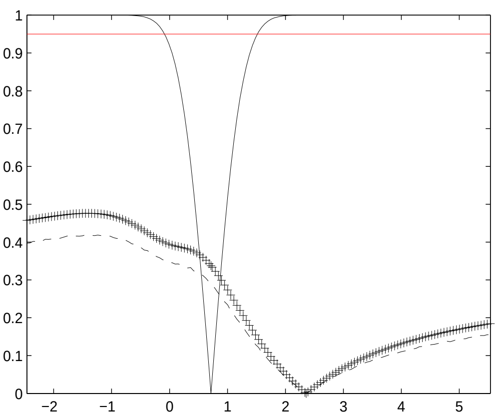
(b) Traded $\triangle I_{Nres}$

(c) Non-traded $\triangle I_{Res}$

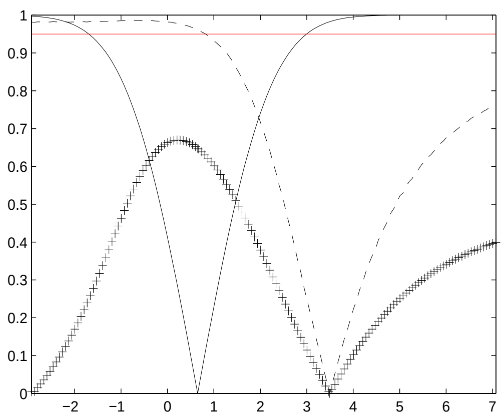
(d) Traded $\triangle I_{Res}$

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

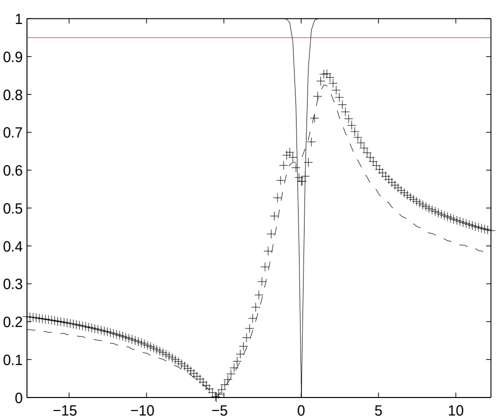
Figure 3: $1-p$ plots for risk premium in Yogo (2006) using 25 FF Portfolios



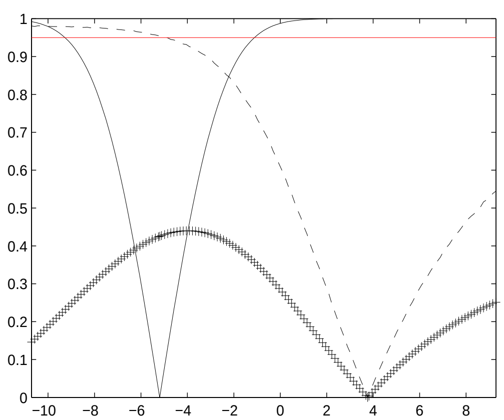
(a) Non-traded $\triangle C_{Ndur}$



(b) Traded $\triangle C_{Ndur}$



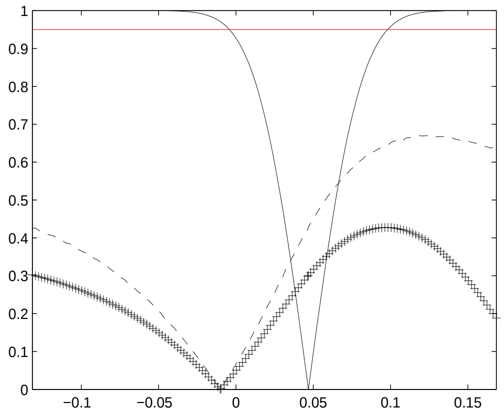
(c) Non-traded $\triangle C_{Dur}$



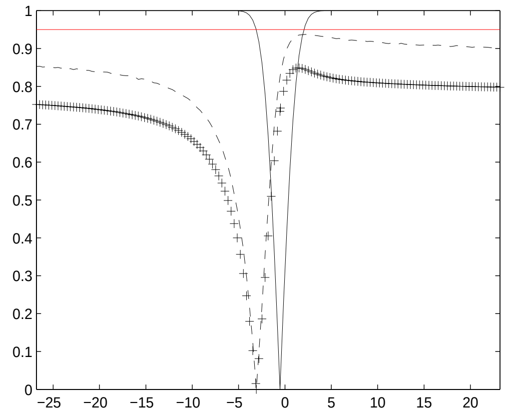
(d) Traded $\triangle C_{Dur}$

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

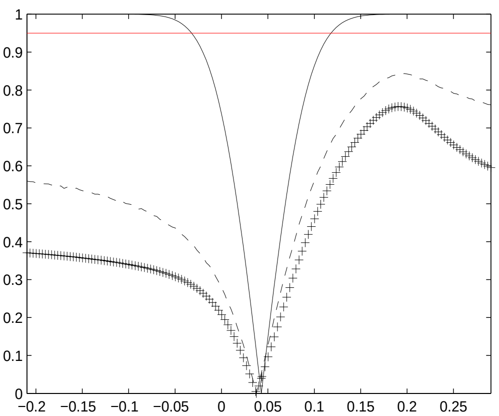
Figure 4: $1 - p$ plots for risk premium in Li-Vassalou-Xing (2006) using 25 FF Portfolios



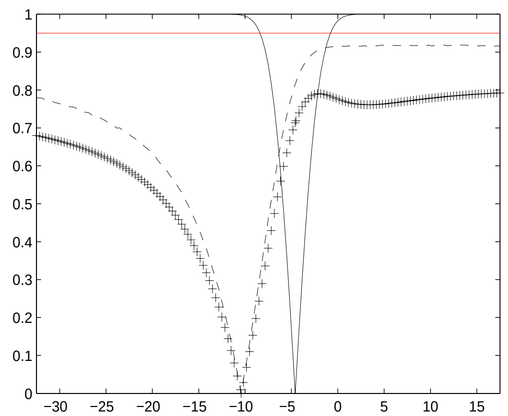
(a) Non-traded $Hholds$



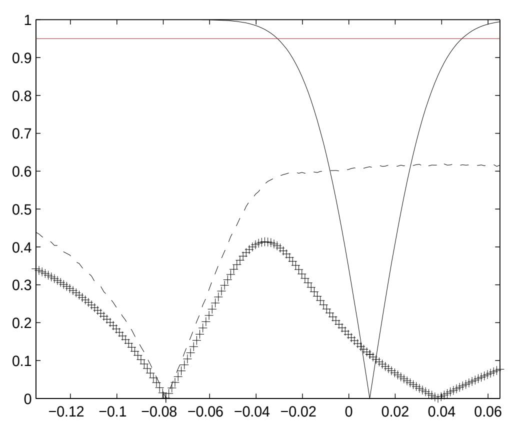
(b) Traded $Hholds$



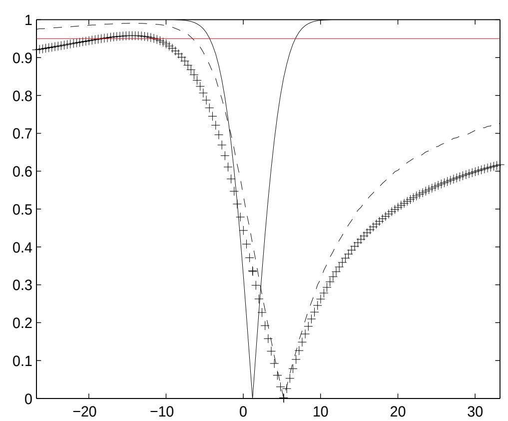
(c) Non-traded $Nfinco$



(d) Traded $Nfinco$



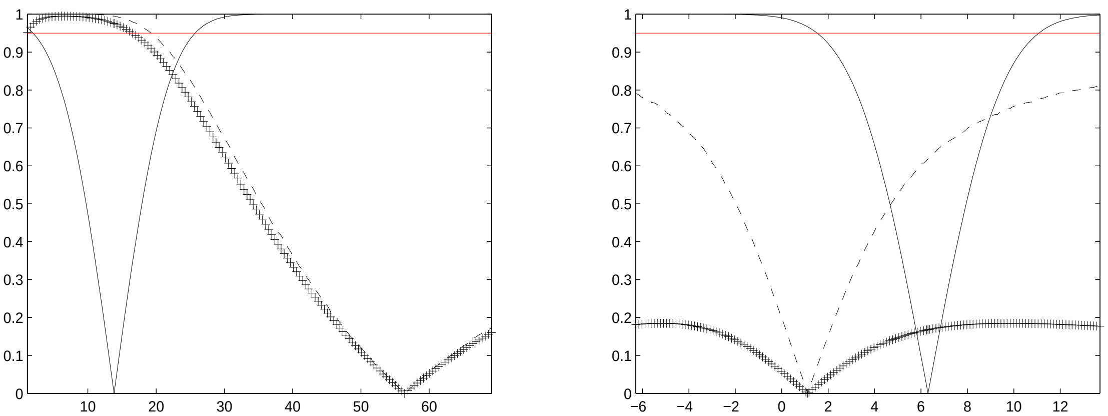
(e) Non-traded $Finan$



(f) Traded $Finan$

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

Figure 5: $1 - p$ plots for risk premium in Muir et al. (2011) using 25 FF Portfolios

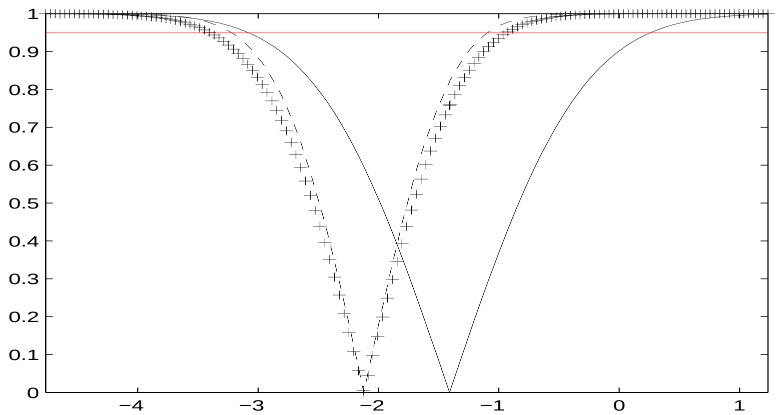


(a) Non-traded *Lev*

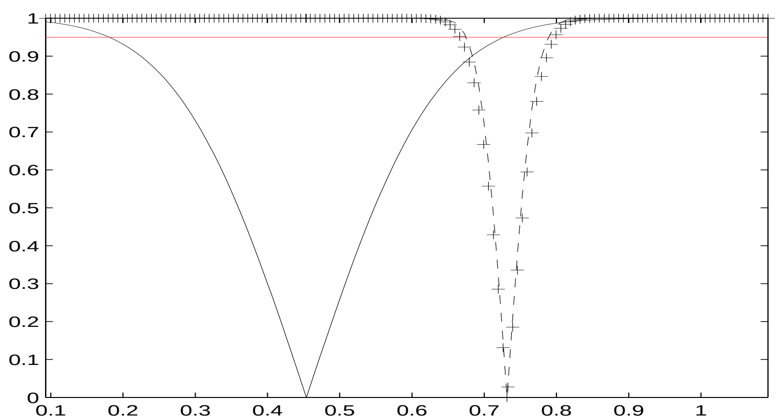
(b) Traded *Lev*

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

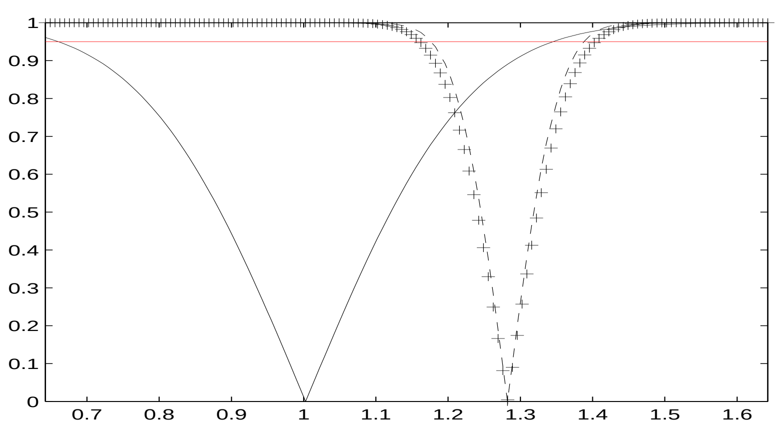
Figure 6: $1 - p$ plots for risk premium in Fama-French (1993) using 25 FF + 30 Industry Portfolios



(a) R_M



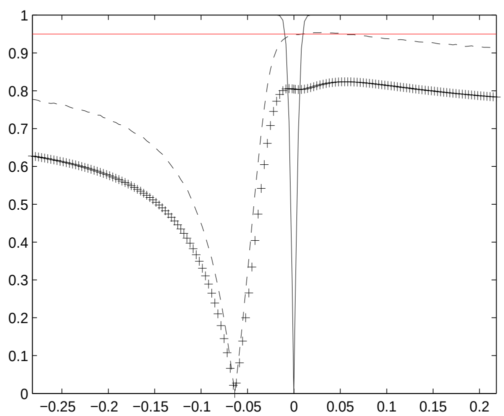
(b) SMB



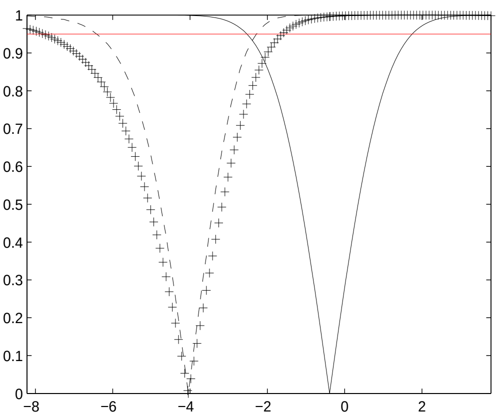
(c) HML

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

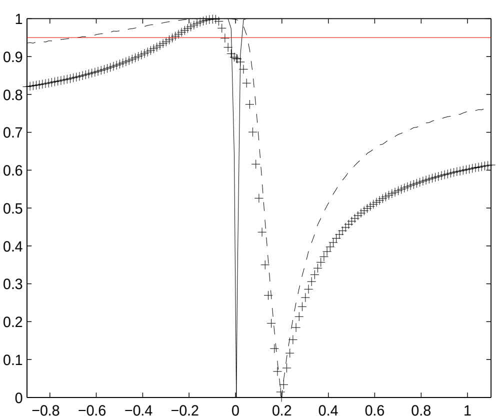
Figure 7: $1-p$ plots for risk premium in Cochrane (1996) using 25 FF + 30 Industry Portfolios



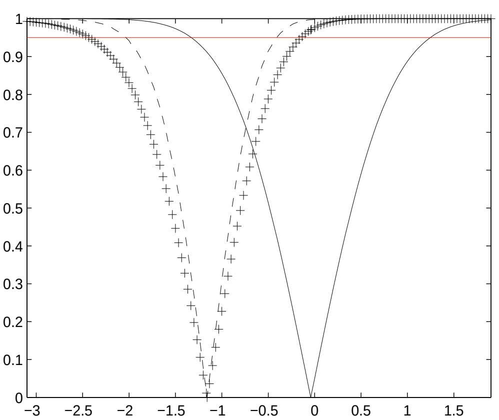
(a) Non-traded $\triangle I_{Nres}$



(b) Traded $\triangle I_{Nres}$



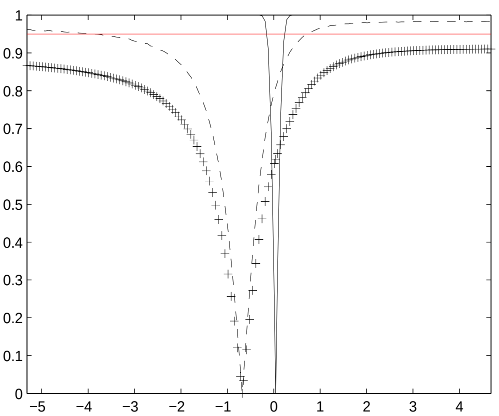
(c) Non-traded $\triangle I_{Res}$



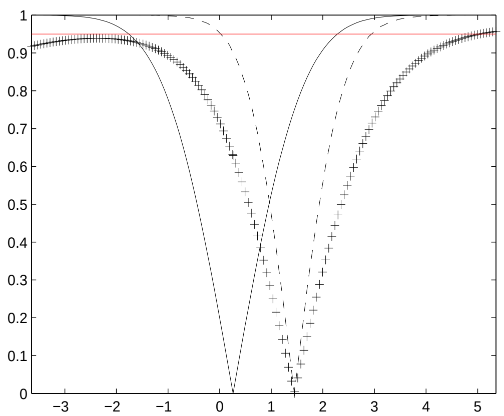
(d) Traded $\triangle I_{Res}$

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

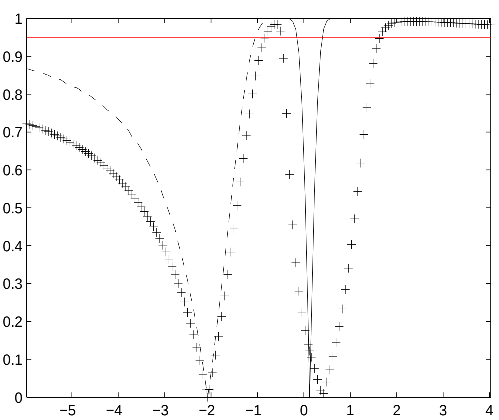
Figure 8: $1 - p$ plots for risk premium in Yogo (2006) using 25 FF + 30 Industry Portfolios



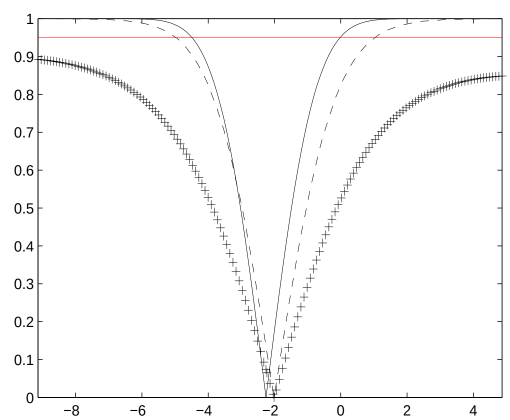
(a) Non-traded $\triangle C_{Ndur}$



(b) Traded $\triangle C_{Ndur}$



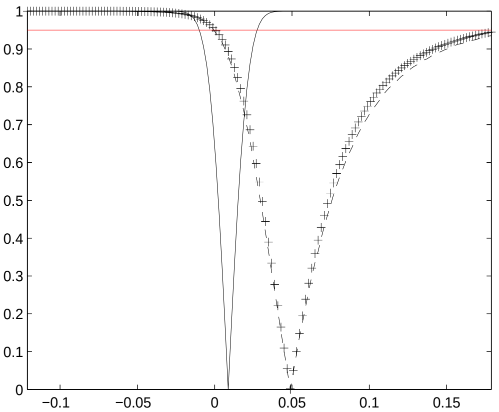
(c) Non-traded $\triangle C_{Dur}$



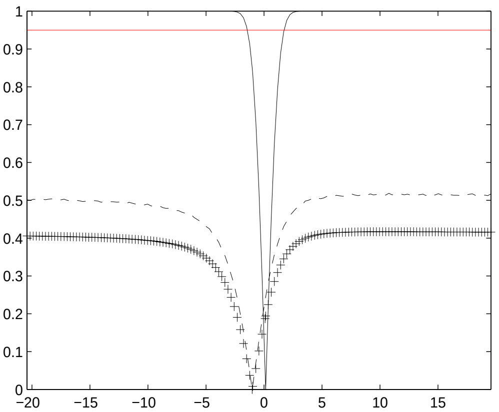
(d) Traded $\triangle C_{Dur}$

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

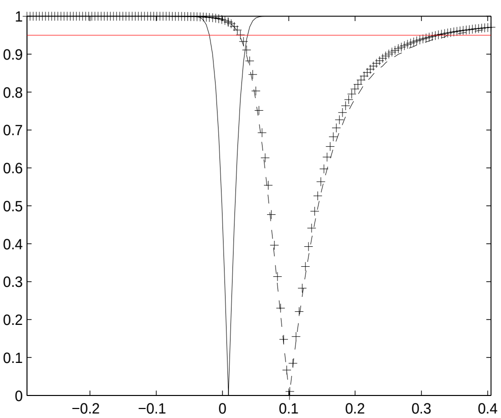
Figure 9: $1-p$ plots for risk premium in Li-Vassalou-Xing (2006) using 25 FF + 30 Industry Portfolios



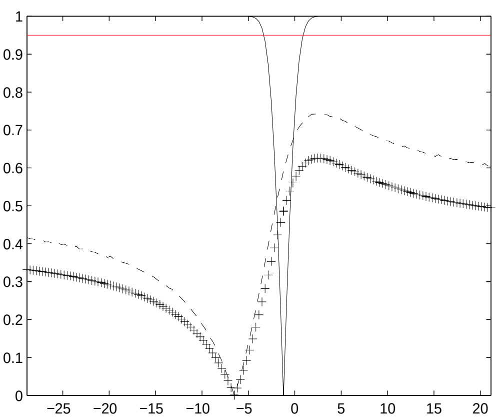
(a) Non-traded *Hholds*



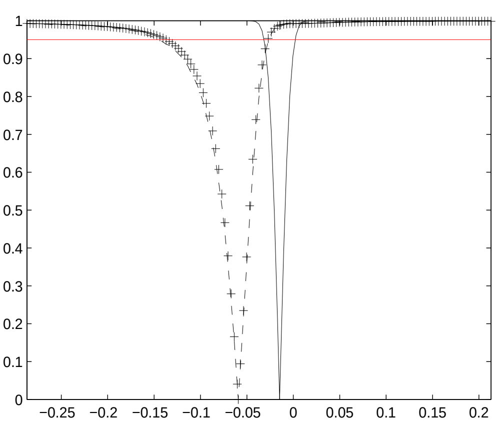
(b) Traded *Hholds*



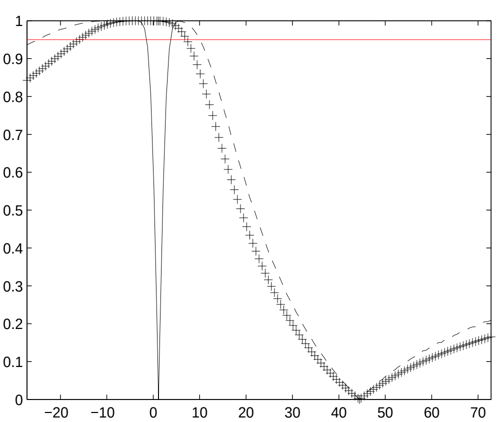
(c) Non-traded *Nfinco*



(d) Traded *Nfinco*



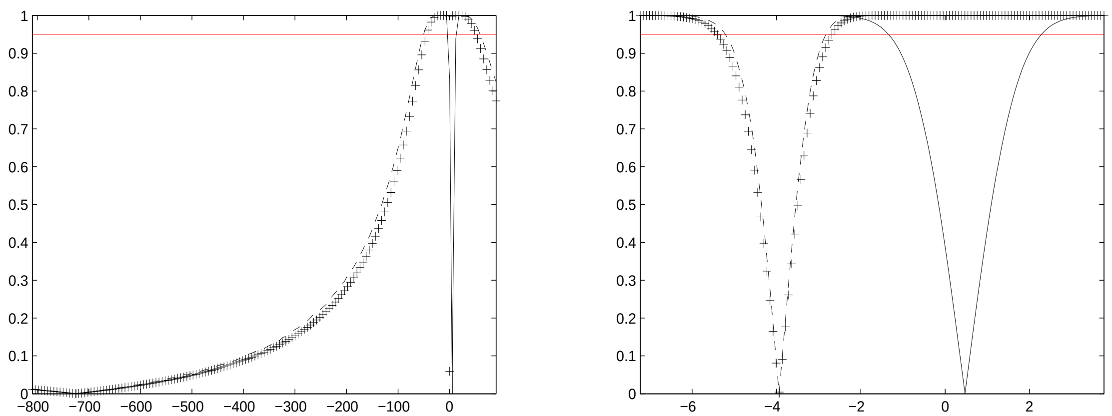
(e) Non-traded *Finan*



(f) Traded *Finan*

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).

Figure 10: $1 - p$ plots for risk premium in Muir et al. (2011) using 25 FF + 30 Industry Portfolios



(a) Non-traded *Lev* (b) Traded *Lev*

Note: FM t-statistic (solid line), FKLM (solid-plusses), FCLR (dashed).