Two Trees with Heterogeneous Beliefs: 
Spillover Effect of Disagreement*

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Abstract

We study a model in which two groups of investors have different beliefs about the expected dividend growth rates of two stocks. The state price density is affected by investor disagreements for both stocks, especially that for the larger stock. The model predicts a positive relation between investor disagreement about one stock and the expected return as well as return volatility of the other stock, even when the fundamentals of two stocks are uncorrelated. This spillover effect is stronger from a large stock to a small stock than vice versa. These novel model implications are supported by empirical analysis.

JEL Classification: D53, G12

Keywords: Heterogenous Beliefs, Disagreement, Multiple Stocks, Size, Expected Return, Volatility
1 Introduction

The traditional asset pricing literature is predominately built on representative agent models. A growing body of research has recently highlighted that investor heterogeneity can play an important role in understanding the behavior of asset prices (see, e.g., Basak (2005) for a recent survey). These studies have focused on the case of a single risky asset and examined how investor disagreement for one firm affects its own stock’s trading, volatility, and pricing. Disagreement about either fundamental or extraneous variables (e.g., dividends, earnings, sunspot, or rare events) can generate higher stock return volatility, larger trading volume and predicts future stock return.

In this paper, we contribute to the heterogenous belief literature by studying stock prices in a Lucas-type exchange economy with multiple risky assets. We extend Cochrane, Longstaff, and Santa-Clara (2008) (hereafter CLS) to an economy in which the two investors have different beliefs about the growth rate of dividend of two Lucas trees.1 Our model features incomplete information about the dynamics of dividend process (growth rate of dividend payment is unknown). Investors have different priors about the dividend growth rates, and Bayesian update their beliefs based on realized dividends. Due to the difference in prior, investors obtain different estimates and they “agree to disagree” about the unobservable dividend growth rates.

We first derive the equilibrium quantities. Using the technique of Cuoco and He (1994) and Basak and Cuoco (1998), we construct a representative agent with the wealth-weighted power utility function. Utilizing Clark-Ocone Malliavine derivatives, we derive the expected stock return and return volatility. Then, we examine the relation between the disagreement

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1 CLS use a two-tree model to explain the serial correlation of stock returns and the predictability of stock returns by price-dividend ratios. Their work is further generalized. Eberly and Wang (2009) extend the CLS setup to a production economy and analyze firm’s investment and Tobin’s Q. In Branger, Schlag, and Wu (2011), the dividend drift is a known constant for one stock, but stochastic for the second stock. Martin (2013) generalizes the CLS setup to an arbitrary number of trees and apply it to study rare disasters. These papers use representative-agent models, while we study the effects of investors disagreement on expected return and volatility in a model with heterogenous investors.
about the fundamental of one stock and the expected return or volatility of another stock. The two stocks differ in their size, or the share each stock’s dividend contributes to the aggregate dividend. We find that there is a “spillover” of the disagreement effect, above and beyond the effect of disagreement on own stock which is the focus of prior research. Investors’ disagreement about the fundamental of one stock would affect the expected return as well as the return volatility of the other stock, even when the fundamentals of the two stocks are uncorrelated. The spillover effect is stronger from the large stock to the small stock than from the small stock to the large stock. In other words, disagreement about a large-size stock has a stronger effect on the expected return and volatility of a small-size stock. For a small stock, the disagreement of other (larger) stock matters more for its expected return and volatility than the disagreement of the own stock.

Our paper has two new theoretical contributions. First, we uncover a positive relation between investors’ disagreement about the fundamental of stock 1 and the return volatility of stock 2, and this effect is stronger when stock 1 is larger in size than stock 2. The intuitions are as follows: the representative agent’s utility is a weighted average of the two investors, and the state price density or pricing kernel depends on the relative weight assigned to the two investors in the representative agent’s utility. This relative weight is stochastic and depends on the investors’ disagreements for both stocks. We show that higher disagreements among investors lead to a more volatile relative weight and hence more volatile pricing kernel, which in turns implies higher stock return volatility. In addition, the state price density also depends on aggregate endowment, whose volatility is proportionally related to the dividend share. Therefore, the stock volatility is also a function of dividend share. Since the state price density is more heavily influenced by investors’ disagreement for the larger stock, such disagreement has a larger impact on the return volatility of the other (smaller) stock. Second, similar intuitions also predict a positive relation between the disagreement of stock 1 and the expected return of stock 2, and this effect is stronger when stock 1 is larger in size than stock 2. These two model predictions distinguish our paper from the existing literature.
We test the two predictions of our model using the dispersion of analyst forecasts as the proxy of investor disagreement. Consistent with our model’s unique prediction, we find a positive and significant relation between a stock’s expected return and the analyst forecast dispersion for the industry leader (the firm in the same industry with the largest market capitalization). We also find a positive and significant relation between the return volatility of a stock and the analyst forecast dispersion for the industry leader. Further confirming the spillover effect of investor disagreement, we sort stocks into 50 portfolios each month based on book-to-market ratio, and find a positive cross-sectional relation between a stock’s return next month and the current dispersion of analyst forecast for the largest stock that belongs to the same book-to-market sorted portfolio.

Our paper contributes to a large literature on heterogeneous beliefs or disagreement on fundamental and nonfundamental variables. Buraschi, Trojani, and Vedolin (2013) and Chabakauri (2013) are closest related to our paper in terms of model setup, but they examine different economic questions. Buraschi, Trojani, and Vedolin (2013) consider an economy with two trees and two investors with different beliefs. They focus on the impact of disagreement on variance and correlation risk premia. They find that investors’ disagreement helps explain the differential pricing of index and individual equity options. Chabakauri (2013) study dynamic equilibrium in a Lucas economy with two stocks, two heterogeneous constant relative risk aversion investors facing margin and leverage constraints. They find a positive relationship between the amount of leverage in the economy and magnitudes of stock return correlations and volatilities. We are the first to study, both theoretically and empirically, the impact of investor disagreement for one stock on the expected return and volatility of another stock. We find that there is a “spillover” of the disagreement effect, and such effect is asymmetric across stocks depending their size.

There is an extensive theoretical literature that implies that difference in beliefs should lead to a positive risk premium (e.g., Varian (1985), Abel (1989), David (2008)). Empirically, Boehme, Danielsen, Kumar, and Sorescu (2009) and Avramov, Chordia, Jostova, and Philipov (2009) document a positive relation between disagreement and expected stock return. However, Chen, Hong, and Stein (2002) and Diether, Malloy, and Scherbina (2002) report a negative relation, which is consistent with the Miller (1977) hypothesis when short-sale constraints are present. Most recently, Carlin, Longstaff, and Matoba (2014) document a positive risk premium for disagreement in the mortgage-backed security market, a liquid essentially free from short-sale constraints. All of these papers analyze the impact of investor disagreement about the fundamental of one risky asset on its own expected return. We examine the effects of disagreement about one stock (especially a large stock) on the expected return and return volatility of other stocks (especially smaller stocks) - a "spillover" effect of disagreement.

The rest of the paper is organized as follows. Section 2 describes the model and solves the dynamic equilibrium. Section 3 numerically illustrates the dispersion spillover effect and derives two testable hypotheses. Empirical test results are presented in Section 4. Section 5 concludes the paper. The Appendix contains the proof of Propositions and a summary of key model parameters and variables.

2 The Model

We extend the two-tree model of CLS to an economy where two investors \((i = A, B)\) have identical preferences and endowments but different beliefs about the expected dividend growth rates of the two Lucas trees (or firms). There is a single consumption good.

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3 Gallmeyer and Hollifield (2008) study the effects of a market-wide short-sale constraint in a dynamic economy with heterogeneous beliefs. They show that imposing the constraint reduces the stock price if the optimistic investors’ intertemporal elasticity of substitution (IES) is less than one and increases the stock price if the optimist’s IES is greater than one.

4 At the portfolio level, Yu (2011) documents that expected market return is negatively related to disagreement measured bottom-up from individual-stock analyst forecast dispersions, arguing that an increase in market disagreement manifests as a drop in discount rate.
(the numeraire) and the economy has a finite horizon \([0, T]\). The uncertainty is generated by a four-dimensional Brownian motion, \(\omega = (\omega_1, \omega_{\mu 1}, \omega_2, \omega_{\mu 2})\) on a filtered probability space \(\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P\right)\). After setting up the model in details, we solve the equilibrium and derive novel model implications on the effects of the disagreement about the expected dividend growth rate of one stock for the expected return and return volatility of the other stock.

### 2.1 Cash Flow and Information Structure

In our economy, the aggregate endowment is composed of two dividend streams, \(\varepsilon = D_1 + D_2\), where \(D_1\) and \(D_2\) denote the dividend generated by trees 1 and 2, respectively. The dividend process \(D_j\) \((j = 1, 2)\) follows a Geometric Brownian motion with stochastic drift given by:

\[
\begin{align*}
    dD_j(t) &= D_j(t) \left[\mu_j(t) dt + \sigma_j d\omega_j(t)\right], \\
    d\mu_j(t) &= \kappa_j (\pi_j - \mu_j(t)) dt + \sigma_{\mu j} d\omega_{\mu j}(t),
\end{align*}
\]

where \(\mu_j\) and \(\sigma_j\) are the expected growth rate and volatility of dividend \(D_j\), respectively. \(\pi_j\) is the long-run mean of expected growth rate, \(\kappa_j > 0\) is the mean-reverting parameter, and \(\sigma_{\mu j}\) denotes the volatility of expected growth rate. The uncertainty vector \(\omega = \left(\omega_1, \omega_{\mu 1}, \omega_2, \omega_{\mu 2}\right)\) is a four-dimensional Brownian motion under the objective probability measure \(P\). Without loss of generality, we assume that the four Brownian motions \(\omega_1, \omega_{\mu 1}, \omega_2, \omega_{\mu 2}\) are uncorrelated.\(^5\)

The investors \((i = A, B)\) observe the realizations of dividend process, \(D_j\), but have incomplete information about the fundamental risk \(\omega_j\). They can exactly estimate \(\sigma_j\) from the quadratic variation of \(D_j\), and update their estimates of \(\mu_j\) after observing the realizations of \(D_j\). These assumptions are standard in the literature of incomplete information and het-

\(^5\)We can consider a general case where the dividend processes of the two stocks are correlated. However, to distinguish the effect of investors’ heterogeneity about dividend growth of one stock on the return of another stock from the correlation effect between the two stocks, we assume the dividend processes of the two stocks are uncorrelated.
erogeneous beliefs (e.g., Detemple and Murthy 1994; Scheinkman and Xiong 2003; Buraschi and Jiltsov 2006; David 2008; Dumas, Kurshev, and Uppal 2009; Xiong and Yan 2010). Following the same literature, we assume that investors have different initial prior beliefs about $\mu_j$, that is, $\mu_j^A(0) \neq \mu_j^B(0)$. Each investor, under his subjective probability measure $P^i$, employs the standard filtering theory (see, e.g., Liptser and Shiryaev 1977) to form his posterior beliefs about $\mu_j$: $\mu_i^j(t) \equiv E^i[\mu_j(t) | \mathcal{F}_t]$, which follows the dynamics given by:

$$d\mu_i^j(t) = \kappa_j (\pi_j - \mu_i^j(t)) dt + \frac{\nu_i^j(t)}{\sigma_j} d\omega_i^j(t), \quad (2)$$

where $\omega_i^j(t) = \frac{1}{\sigma_j} \int_0^t \left( \frac{dD_j(s)}{D_j(s)} - \mu_j^i(s) ds \right)$ is investor $i$’s innovation process of $D_j$ under his probability measure, and $\nu_i^j(t) \equiv E^i \left[ (\mu_i^j(t) - \mu_j(t))^2 \right]$ is the conditional variance of investor $i$’s belief about $\mu_j$. The standard filtering theory implies that $\nu_i^j$ is a deterministic function:

$$\frac{d\nu_i^j(t)}{dt} = \sigma_j^2 - 2\kappa_j \nu_i^j(t) - \left( \frac{\nu_i^j(t)}{\sigma_j} \right)^2. \quad (3)$$

**Remark 1 (Learning)** We assume that investors have identical prior conditional variance of their beliefs about $\mu_j$ (see, e.g., Basak 2005), that is, $\nu_j^A(0) = \nu_j^B(0) \equiv \nu_j(0)$. Under this assumption, the conditional variance of the belief about $\mu_j$ is the same for the two investors at any time: $\nu_j^A(t) = \nu_j^B(t) \equiv \nu_j(t) \ (\forall t)$. The stationary conditional variance of the belief is $\nu_j = \left( \sqrt{\kappa_j^2 + (\sigma_{\mu j}/\sigma_j)^2 - \kappa_j} \right) \sigma_j^2$ which satisfies $\frac{d\nu_j(t)}{dt} = 0$ (e.g., Scheinkman and Xiong 2003; Buraschi and Jiltsov 2006; Dumas, Kurshev, and Uppal 2009; Xiong and Yan 2010).

By replacing $\nu_j(t)$ with $\nu_j$ in Equation (2), the dynamics of investor $i$’s belief about the expected dividend growth rate, $\mu_i^j$, becomes

$$d\mu_i^j(t) = \kappa_j (\pi_j - \mu_i^j(t)) dt + \frac{\nu_j}{\sigma_j^2} d\omega_i^j(t), \quad (4)$$

which means that investor $i$’s posterior estimate of $\mu_j$ follows a mean-reverting process with the inverse precision of $\frac{\nu_j}{\sigma_j^2}$.

By manipulating investor $i$’s perceived shocks to the dividend growth rate $\omega_i^j(t)$, we derive
its dynamics given by:

$$d\omega_j(t) = \frac{1}{\sigma_j} \left( \frac{dD_j(t)}{D_j(t)} - \mu_j(t) dt \right) = d\omega_j(t) + \frac{\mu_j(t) - \mu_j(t)}{\sigma_j} dt,$$  \hspace{1cm} (5)$$

and thus the relation between two investors’ perceived innovations to dividend growth rates is given by:

$$d\omega_j^B(t) = d\omega_j^A(t) + g_j(t) dt,$$  \hspace{1cm} (6)$$

where investor disagreement $g_j$ is defined as the (normalized) difference in beliefs about expected dividend growth $\mu_j$:

$$g_j(t) = \frac{\mu_j^A(t) - \mu_j^B(t)}{\sigma_j}$$

Its dynamics satisfies:

$$dg_j(t) = \frac{1}{\sigma_j} (d\mu_j^A(t) - d\mu_j^B(t)) = -\phi_j g_j(t) dt,$$  \hspace{1cm} (7)$$

where $\phi_j = \frac{v_j}{\sigma_j^2} + \kappa_j > 0$. Equation (7) implies that $g_j$ is a deterministic function,\(^6\) satisfying:

$$g_j(t) = g_j(0) \frac{\sigma_j^2}{\sigma_j^2 + v_j t}.$$  \hspace{1cm} (6a)$$

Therefore, if investor A is more optimistic at time $t = 0$, that is, $g_j(0) > 0$, he will remain more optimistic than investor B over time. The disagreement $g_j$ plays a key role for state price density (Equation(25)), stock valuation (Equation(30)) as well as in determining the expected stock return and return volatility (Equation(33)).

Applying the Ito’s lemma, the aggregate endowment $\varepsilon = D_1 + D_2$ follows a stochastic process given by:

$$d\varepsilon(t) = \varepsilon(t) \left[ (f(t) \mu_1(t) + (1 - f(t)) \mu_2(t)) dt + f(t) \sigma_1 d\omega_1(t) + (1 - f(t)) \sigma_2 d\omega_2(t) \right],$$  \hspace{1cm} (8)$$

Under the objective probability measure (the first line), the drift term of the aggregate endowment is the weighed-average of expected growth rates of dividends 1 and 2, where

\(^6\)If the two investors have different initial conditional variance of their beliefs, the stationary variance will be different for two investors and then the disagreement will follow a stochastic process (e.g., Buraschi and Jiltsov 2006).
the weight is the share of the corresponding tree’s dividend in the aggregate endowment. There are two components in the diffusion term: the first is the product of the share of dividend 1, \( f = \frac{D_1}{D_1 + D_2} \), by its volatility, \( \sigma_1 \), and the second is the product of the share of dividend 2, \( (1 - f) \), by its volatility, \( \sigma_2 \). The second line describes the dynamics of the aggregate endowment from the perspective of investor \( i \) \( (i = A, B) \) by manipulating the relation between \( d\omega_j(t) \) and \( d\omega_j(t) \).

Since dividend process \( D_1 \) and aggregate endowment process \( \varepsilon \) are stochastic, the dividend share of the first tree, \( f \), follows a stochastic process given by:

\[
\begin{align*}
    df(t) &= f(t)(1 - f(t)) \left[ \mu_1(t) - \mu_2(t) - f(t)\sigma_1^2 + (1 - f(t))\sigma_2^2 \right] dt \\
    &\quad + f(t)(1 - f(t)) \left( \sigma_1 d\omega_1(t) - \sigma_2 d\omega_2(t) \right).
\end{align*}
\]

After a positive (resp. negative) shock to tree 1’s dividend, its share will increase (resp. decrease).

Since investors have incomplete information about fundamental risk \( \omega_j \) and learn about \( \mu_j \) over time, they have different perception about the dynamics of dividend share too. From the perspective of investor \( i \), the share of first dividend follows the dynamics:

\[
\begin{align*}
    df(t) &= f(t)(1 - f(t)) \left[ \mu_1^i(t) - \mu_2^i(t) - f(t)\sigma_1^2 + (1 - f(t))\sigma_2^2 \right] dt \\
    &\quad + f(t)(1 - f(t)) \left( \sigma_1 d\omega_1^i(t) - \sigma_2 d\omega_2^i(t) \right).
\end{align*}
\]

### 2.2 Investment Opportunities

Investors face two fundamental risks \( \omega_1 \) and \( \omega_2 \) in the economy, which are related to the two dividend processes \( D_1 \) and \( D_2 \), respectively. To make the market (dynamically) complete, investors continuously trade in three assets: one riskless asset (bond) in zero net supply, and two stocks 1 and 2 (claims on the two dividends \( D_1 \) and \( D_2 \) respectively), each in net supply of one unit. The bond price, \( B \), and the prices of two stocks \( S_1 \) and \( S_2 \) follow the stochastic
processes given by:

\[
\frac{dB(t)}{B(t)} = r(t) \, dt,
\]

\[
\frac{dS_1(t)}{S_1(t)} = \mu_{S_1}(t) \, dt + \sigma_{S_11}(t) \, d\omega_1(t) + \sigma_{S_12}(t) \, d\omega_2(t)
\]

\[
= \mu^i_{S_1}(t) \, dt + \sigma_{S_11}(t) \, d\omega^i_1(t) + \sigma_{S_12}(t) \, d\omega^i_2(t),
\]

\[
\frac{dS_2(t)}{S_2(t)} = \mu_{S_2}(t) \, dt + \sigma_{S_21}(t) \, d\omega_1(t) + \sigma_{S_22}(t) \, d\omega_2(t)
\]

\[
= \mu^i_{S_2}(t) \, dt + \sigma_{S_21}(t) \, d\omega^i_1(t) + \sigma_{S_22}(t) \, d\omega^i_2(t),
\]

where the objective expected return and return volatility of two stocks are endogenously determined in equilibrium. Investors observe the historical prices of two stocks and have the same estimates about their volatilities due to the quadratic variation-covariance of \(S_1\) and \(S_2\), but they have different estimates of \(\mu_{S_j}\) since they have different perceptions about the innovation \(\omega_j\). Investors A’s and B’s subjective expected stock returns are denoted by \(\mu^A_{S_j}\) and \(\mu^B_{S_j}\), respectively. From Equation (11), we can derive the following relation between the subjective and objective expected stock returns for investor \(i = A, B\):

\[
\mu^i_{S_1}(t) - \mu_{S_1}(t) = \sigma_{S_11}(t) \left( \frac{\mu^i_1(t) - \mu_1(t)}{\sigma_1} \right) + \sigma_{S_12}(t) \left( \frac{\mu^i_2(t) - \mu_2(t)}{\sigma_2} \right),
\]

\[
\mu^i_{S_2}(t) - \mu_{S_2}(t) = \sigma_{S_21}(t) \left( \frac{\mu^i_1(t) - \mu_1(t)}{\sigma_1} \right) + \sigma_{S_22}(t) \left( \frac{\mu^i_2(t) - \mu_2(t)}{\sigma_2} \right),
\]

By the definition of the disagreement \(g_j\) between the two investors for stock \(j = 1, 2\), Equation (14) implies that the differences in the subjective expected stock returns between the two investors depend on the dispersion in beliefs about dividend growth rate:

\[
\mu^A_{S_1}(t) - \mu^B_{S_1}(t) = \sigma_{S_11}(t) \, g_1(t) + \sigma_{S_12}(t) \, g_2(t),
\]

\[
\mu^A_{S_2}(t) - \mu^B_{S_2}(t) = \sigma_{S_21}(t) \, g_1(t) + \sigma_{S_22}(t) \, g_2(t).
\]

Note that difference in the subjective expected return of one stock arises not just due to investors’ disagreement about its own fundamental, but also because of their disagreement
about the fundamental of the other stock.

Because there are three traded assets and two fundamental risks in the economy, the market is dynamically complete under investor $i$’s subjective probability measure $P^i$. There exists an individual-specific state-price density $\xi^i$ for investor $i$, which satisfies the following dynamics:

$$\begin{align*}
    d\xi^i (t) = -\xi^i (t) \left( r (t) dt + \theta_1^i (t) d\omega^i_1 (t) + \theta_2^i (t) d\omega^i_2 (t) \right),
\end{align*}$$

(16)

where $r$ is the riskfree rate, and $\theta_j^i$ is the market price of fundamental risk $j$ perceived by investor $i$ satisfying that:

$$\begin{align*}
    \begin{pmatrix}
        \mu^i_{S1} (t) - r (t) \\
        \mu^i_{S2} (t) - r (t)
    \end{pmatrix} &= \begin{pmatrix}
        \sigma_{S11} (t), & \sigma_{S12} (t) \\
        \sigma_{S21} (t), & \sigma_{S22} (t)
    \end{pmatrix} \begin{pmatrix}
        \theta_1^i (t) \\
        \theta_2^i (t)
    \end{pmatrix},
\end{align*}$$

(17)

which means that the equity premium perceived by investor $i$ is related to the market price of two fundamental risks perceived by him, $\theta_1^i$ and $\theta_2^i$.

By manipulating Equations (15) and (17), we find that the differences in the market price of fundamental risks perceived by investors are equal to investor disagreement, i.e., dispersion of beliefs about the corresponding dividend growth rate scaled by dividend volatility:

$$\begin{align*}
    \theta_1^i (t) - \theta_1^j (t) &= g_1 (t), \\
    \theta_2^i (t) - \theta_2^j (t) &= g_2 (t).
\end{align*}$$

(18)

We will derive explicit expressions for the market prices of fundamental risks $\theta_j^i$ in Equation (27).

### 2.3 Investors’ Optimization Problem

At initial time $t=0$, investor $i$ ($A, B$) is endowed with $\alpha_j^i > 0$ share of stock $j$ ($= 1, 2$) such that $\alpha_j^A + \alpha_j^B = 1$, and thus the initial wealth of investor $i$ is $W^i (0) = \alpha_1^i S_1 (0) + \alpha_2^i S_2 (0)$.

Investor $i$ chooses a nonnegative consumption process $c^i$ and portfolio $\pi^i = (\pi_B^i, \pi_{S1}^i, \pi_{S2}^i)$ to maximize his lifetime expected utility of consumption, $E_t^i \left[ \int_0^T e^{-\beta t} u^i (c^i (t)) dt \right]$, subject to intertemporal budget constraints. The wealth of investor $i$ satisfies $W^i (T) > 0$ and follows
the dynamics:

\[ dW^i (t) = (W^i (t) r (t) - \dot{c}^i (t)) \, dt \]
\[ + \pi^i_{S1} (t) \left[ (\mu^i_{S1} (t) - r (t)) \, dt + \sigma_{S11} (t) \, d\omega^i_1 (t) + \sigma_{S12} (t) \, d\omega^i_2 (t) \right] \]
\[ + \pi^i_{S2} (t) \left[ (\mu^i_{S2} (t) - r (t)) \, dt + \sigma_{S21} (t) \, d\omega^i_1 (t) + \sigma_{S22} (t) \, d\omega^i_2 (t) \right]. \] (19)

Using the martingale techniques (e.g., Cox and Huang 1987; Karatzas, Lehoczky, and Shreve 1987), each investor’s dynamic optimization problem can be recast into a static optimization problem (e.g., a static budget constraint), that is,

\[ \max_{c^i (.) , \pi^i (.)} E_i \left[ \int_0^T e^{-\beta t} u^i (c^i (t)) \, dt \right] \quad \text{s.t.} \quad \int_0^T \xi^i (t) \, c^i (t) \, dt \leq W^i (0), \] (20)

where \( u^i (c^i (.) ) = \frac{c^i (.)^{1-\gamma}}{1-\gamma} \). Investors have the same coefficient of risk aversion, \( \gamma \) and time discount factor, \( \beta \), but have different prior beliefs about the expected dividend growth \( \mu_j \). \( E_i \) denotes the time-\( t \) conditional expectation operator under investor \( i \)'s probability measure.

The static budget constraint implies that the present value of investor \( i \)'s lifetime consumption stream can not exceed his initial wealth \( W^i (0) \). By solving investor \( i \)'s optimal problem, the first-order condition for consumption \( c^i (t) \) implies

\[ (u^i)' (c^i (.) ) = y^i \xi^i (t), \] (21)

where \( \xi^i \) is the state-price density from the perspective of investor \( i \) which will be determined in Equation (24), and \( y^i \) the Lagrangian multiplier for investor \( i \)'s static budget constraint. At the optimum consumption, investor \( i \)'s static budget constraint holds with equality, that is, \( \int_0^T \xi^i (t) \, c^i (t) \, dt = W^i (0) \).

2.4 Equilibrium

Definition 1 (Equilibrium) An equilibrium is a price system \((r, S_1, S_2)\) and consumption-portfolio processes \((c^i, \pi^i)\) \((i = A, B)\) such that: (i) investors choose their optimal consumption-
portfolio strategies given the price system \((r, S_1, S_2)\). (ii) the security price processes are consistent across investors. (iii) the goods and two stock markets clear, i.e.,

\[
c^A(t) + c^B(t) = \varepsilon(t),
\]

\[
\pi^A_{s_1}(t) + \pi^B_{s_1}(t) = 1,
\]

\[
\pi^A_{s_2}(t) + \pi^B_{s_2}(t) = 1,
\]

where \(\pi^i_{s_j}\) is the portfolio holdings (in percentage) of stock \(j\) by investor \(i\).

Following the technique of Cuoco and He (1994) and Basak and Cuoco (1998), we introduce a “representative” investor with the following utility function defined by:

\[
U(c; \lambda) = \max_{c^A + c^B = c} \left[ u^A(c^A(t)) + \lambda(t) u^B(c^B(t)) \right],
\]

where \(\lambda(.) > 0\) is a state-dependent weight describing the relative importance of investor \(B\) in the representative agent’s utility.

In equilibrium, the market of consumption goods clears and each investor solves his optimal problem (Equation (21)). The representative investor consumes the aggregate endowment and his optimal solution associated with Equation (23) implies that

\[
\lambda(t) = \frac{(u^A)'(c^A(t))}{(u^B)'(c^B(t))} = \frac{y^A\xi^A(t)}{y^B\xi^B(t)},
\]

Proposition 1 summarizes the equilibrium results.

**Proposition 1 (Equilibrium)** In equilibrium, the state-price of density perceived by investor \(i\) is

\[
\xi^A(t) = \frac{1}{y^A} \left( \frac{1 + \lambda(t)^{1/\gamma}}{\varepsilon(t)} \right)^{\gamma}, \xi^B(t) = \frac{1}{y^B\lambda(t)} \left( \frac{1 + \lambda(t)^{1/\gamma}}{\varepsilon(t)} \right)^{\gamma},
\]

where the relative weight \(\lambda(t)\) of investor \(B\) follows the stochastic process given by:

\[
\frac{d\lambda(t)}{\lambda(t)} = -g_1(t) d\omega^A_1(t) - g_2(t) d\omega^A_2(t),
\]
where \( y^i \) is the Lagrangian multiplier for investor \( i \)'s static budget constraint and such that investor \( i \)'s static budget constraint holds with equality, that is, \( \int_0^T \xi^i(t) c^i(t) \, dt = W^i(0) \).

The optimal consumption is given by

\[
c^A(t) = (1 - \psi(t)) \varepsilon(t), c^B(t) = \psi(t) \varepsilon(t),
\]

where \( \psi(t) = \frac{\lambda(t)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \) is the consumption share of investor \( B \).

Equation (24) implies that investors have different state-price densities and the individual-specific state-price density is driven by the relative weight of two investors. The state-price density \( \xi^i \) is related to market prices of risks perceived by investor \( i \), \( \theta^i_1 \) and \( \theta^i_2 \), and disagreements \( g_1 \) and \( g_2 \).

Equation (25) characterizes the dynamics of the relative weight of investor \( B \). The relative weight in the economy is driven by the two disagreements \( g_1 \) and \( g_2 \), and the two fundamental risks perceived by investor \( A \), \( d\omega^A_1 \) and \( d\omega^A_2 \). The volatility of the relative weight of investor \( B \) increases with investors’ disagreement about expected dividend growth rate. This is intuitive: without loss of generality suppose \( B \) is more pessimistic about the expected growth rate of dividend 1 than investor \( A \), then he will invest less in stock 1. After a positive (negative) shock to the dividend growth of stock 1, investor \( B \) underperforms (outperforms) and thus his relative weight in the economy decreases (increases). The bigger the investor disagreement \( g_1 \) is, the more the relative weight of investor \( B \) would fluctuate, depending on whether the dividend growth of stock 1 experiences a positive or negative shock. Therefore, the volatility of relative weight of investor \( B \) increases with the disagreement \( g_1 \). Same argument can be applied to analyze the effect of disagreement \( g_2 \) on relative weight of investor \( B \).

Equation (26) implies that the consumption of investor \( B \) increases with his relative weight in the economy and the aggregate endowment. The intuition is as follows: (1) a positive shock to dividend leads to a higher aggregate endowment, investors will expect that
there are more goods to consume in the future; and (2) a positive fundamental shock reduces the wealth of investor B (again assuming B is more pessimistic) relative to A, and thus not only his relative weight in the economy decreases but also he expects less goods available for future consumption. Therefore, investor B’s consumption become more volatile with the disagreements of $g_1$ and $g_2$.

Proposition 1 provides equilibrium results of individual-specific state-price density $\xi^i$, which plays an important role in pricing stocks. Applying Ito’s lemma to $\xi^i$ (Equation (24)) and equating its drift and diffusion coefficients to those of Equation (16), we can characterize riskfree rate and market prices of two dividend shocks. The results are presented in Proposition 2.

**Proposition 2 (Market Prices of Fundamental Risks and Riskfree Rate)** Investor’s individual-specific market prices of fundamental risks, $\theta^i_j (t)$ ($i = A, B$; $j = 1, 2$), are given by:

\begin{align}
\theta^A_1 (t) &= \gamma f(t) \sigma_1 + \psi(t) g_1(t), \quad \theta^B_1 (t) = \gamma f(t) \sigma_1 - (1 - \psi(t)) g_1(t), \\
\theta^A_2 (t) &= \gamma (1 - f(t)) \sigma_2 + \psi(t) g_2(t), \quad \theta^B_2 (t) = \gamma (1 - f(t)) \sigma_2 - (1 - \psi(t)) g_2(t),
\end{align}

(27)

and the riskfree interest rate is equal to

\begin{align}
r(t) &= \beta + \gamma \left[ f(t) ((1 - \psi(t)) \mu^A_1 (t) + \psi(t) \mu^B_1 (t)) + (1 - f(t)) ((1 - \psi(t)) \mu^A_2 (t) + \psi(t) \mu^B_2 (t)) \right] \\
&\quad - \frac{\gamma(\gamma+1)}{2} \left[ (f(t) \sigma_1)^2 + ((1 - f(t)) \sigma_2)^2 \right] + \frac{1}{2} \frac{\gamma-1}{\gamma} \psi(t) (g_1(t)^2 + g_2(t)^2). \tag{28}
\end{align}

The market price of fundamental risk for tree 1, $\theta^i_1 (t)$, is composed of two components. The first component is standard and is equal to the product between the coefficient of risk aversion, $\gamma$, and the volatility of aggregate endowment caused by fundamental risk 1, $f(t) \sigma_1$. This is different from the case of one-tree model where the aggregate endowment equals the dividend and then the volatility of endowment equals the volatility of dividend. In the two-
tree model, the volatility of aggregate endowment has two items, \( f(t) \sigma_1 \) and \( (1 - f(t)) \sigma_2 \), which capture the share-weighted volatility of each dividend.

The second component in the expression for the market price of risk is the product of the other investor’s consumption share and the disagreement about fundamental risk 1. Thus, when an investor consumes a smaller share of aggregate endowment, he faces greater exposure to price moving in the direction of the other investor’s beliefs. The intuition for the effect of investor disagreement on the market price of fundamental risk 1 is as follows: when investors have different beliefs about fundamental risk 1, risk is transferred from the more pessimistic investor to the more optimistic who hold more shares of stock 1. The second component is positive for optimistic investor \( \psi(t) g_1(t) \) since he bears more risk, while it is negative for pessimistic investor \( -(1 - \psi(t)) g_1(t) \) since the risk is transferred to optimistic investor and thus he faces less risk. Using the same economic mechanism, we can analyze the market price of fundamental risk 2, \( \theta^2_i(t) \).

The riskfree interest rate is composed of four components. The first item captures investors’ time preference (e.g., time discount factor \( \beta \)). The second item is the dividend-share average of the consumption-share average of investors’ estimates about the expected growth rate of two dividends. This term reflects the wealth effects on consumption. The third item captures the effect of investors’ precautionary saving: as the volatility of aggregate endowments, \( (f(t)\sigma_1)^2 + ((1 - f(t)) \sigma_2)^2 \), increases, investors will face more uncertainty and thus have more incentive to save for future consumption, and reduces the equilibrium interest rate. The last item captures the effect of disagreement: when investors disagree more about the expected dividend growth rate, the two effects arise: (1) substitution effect: investors’ consumption becomes more volatile and thus they have higher precautionary savings motive, therefore the interest rate decreases; and (2) wealth effect: when a positive shock to dividend 1 or 2 occurs, the more optimistic investor’s relative wealth increases and thus his relative weight in the economy also increases, leading to a higher expected dividend growth rate by the representative investor. Expectation of more goods available for future consumption
leads to more consumption today and higher current interest rate.

As a special case, if we set \( f = 1 \), the two-tree model becomes the one-tree model and thus the riskfree rate becomes (e.g., Basak 2000; David 2008)

\[
 r (t) = \beta + \gamma \left( (1 - \psi (t)) \mu^A_1 (t) + \psi (t) \mu^B_1 (t) \right) - \frac{\gamma (\gamma + 1)}{2} \sigma^2 + \frac{1}{\gamma} \psi (t) \left( g_1 (t)^2 + g_2 (t)^2 \right). \tag{29}
\]

In our two-tree model with heterogenous beliefs, both investor disagreements \( g_j \) and the dividend share \( f \) influence the individual-specific market prices of fundamental risks and state-price density. First, the disagreements \( g_1 \) and \( g_2 \) affect the diffusion coefficients of relative weight \( \lambda \), which in turn affects the state-price density \( \xi^A \). Second, the dividend share \( f \) affects the diffusion coefficients of aggregate endowment \( \varepsilon \) (Equation (8)) and thus affects the state-price density \( \xi^A \). Therefore, \( g_j \) and \( f \) should both affect the stock prices, the expected stock returns and return volatilities. Proposition 3 summarizes the results. The proof, based on applications of the Mallavin calculus and the Clark-Ocone formula, can be found in the Appendix.

**Proposition 3 (Stock Price, Stock Volatility, and Expected Stock Return)** The price of stock \( j \), \( S_j \), is\(^7\)

\[
 S_j (t) = \frac{\varepsilon (t)^\gamma}{(1 + \lambda (t)^{1/\gamma})} \mathbb{E}_t^A \left[ \int_t^T e^{-\beta (s-t)} \left( \frac{\lambda (s)^{1/\gamma}}{\varepsilon (s)} \right)^\gamma D_j (s) \, ds \right], \tag{30}
\]

the drift terms for the dynamics of the stock prices under objective probability measure in

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\(^7\)In equilibrium, the stock price can be calculated under the probability measure of either investor. To be consistent with the dynamics of relative weight, we calculate the stock price from the perspective of investor \( A \) (the stock price is the same if we calculate from the perspective of investor \( B \)).
Equations (12) and (13) are given by:

\[
\mu_{S_j}(t) = r(t) + \gamma \left[ f(t) \sigma_1 \sigma_{Sj1}(t) + (1 - f(t)) \sigma_2 \sigma_{Sj2}(t) \right] \\
+ \frac{\sigma_{Sj1}(t)}{\sigma_1} \left[ \mu_1(t) - \left[ (1 - \psi(t)) \mu^A_1(t) + \psi(t) \mu^B_1(t) \right] \right] \\
+ \frac{\sigma_{Sj2}(t)}{\sigma_2} \left[ \mu_2(t) - \left[ (1 - \psi(t)) \mu^A_2(t) + \psi(t) \mu^B_2(t) \right] \right]. \tag{31}
\]

and the diffusion terms for the stock prices in Equations (12) and (13) are given by:

\[
\sigma_{S11}(t) = \sigma_1 + \theta^A_1(t) - \frac{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_1(s) \left[ \psi(s) g_1(t) + \gamma f(s) \sigma_1 + (1 - f(s))(1 - e^\kappa_1(t-s)) \right] ds}{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_1(s)ds}, \tag{32}
\]

\[
\sigma_{S12}(t) = \theta^A_2(t) - \frac{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_1(s) \left[ \psi(s) g_2(t) + \gamma (1 - f(s)) \left( \sigma_2 + \left( 1 - e^\kappa_2(t-s) \right) v_2 \right) \right] ds}{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_1(s)ds},
\]

and

\[
\sigma_{S21}(t) = \theta^A_1(t) - \frac{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_2(s) \left[ \psi(s) g_1(t) + \gamma f(s) \left( \sigma_1 + \left( 1 - e^\kappa_1(t-s) \right) \right) \right] ds}{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_2(s)ds}, \tag{33}
\]

\[
\sigma_{S22}(t) = \sigma_2 + \theta^A_2(t) - \frac{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_2(s) \left[ \psi(s) g_2(t) + \gamma (1 - f(s)) \sigma_2 + (1 - f(s))(1 - e^\kappa_2(t-s)) \right] ds}{E^A_1 \int_t^T e^{-\beta(s-t)} \xi^A(s)D_2(s)ds}.
\]

Note that although we assume that the two fundamental risks are uncorrelated, in equilibrium, the return volatility of one stock is also affected by fundamental risk of the other tree, because the state-price density, via its dependence on the stochastic relative weight of the two investors, is affected by two fundamental risks (see Equations (24) and (25)). In addition, because the relative weight is a function of dispersion of beliefs for both trees, it explains why one stock’s return volatility depends on investor disagreement about its own fundamental as well as investor disagreement about the other stock. Equation (8) implies that the volatility of aggregate endowment is related to dividend share \( f \) and the aggregate
endowment $\varepsilon$ affects the state-price density. Therefore, the stock volatility is also a function of $f$. In Section 3, we numerically analyze the effects of disagreements $g_1$ and $g_2$ and dividend share $f$ on stock volatility.

The subjective expected return of stock $j$ from the perspective of investor $i$ is related to the market prices of two fundamental risks (see (17)):

$$
\mu^i_{S_j}(t) = r(t) + \sigma_{S_{1j}}(t) \theta^i_1(t) + \sigma_{S_{2j}}(t) \theta^i_2(t).
$$

By manipulating the relation between subjective expected stock return $\mu^i_{S_j}(t)$ and objective expected return $\mu_{S_j}(t)$ using Equation (14), we can derive the objective expected stock return given in Equation (31). The risk premium of a stock under the objective probability measure comprises of three components. The first component is equal to the product of risk aversion coefficient and the covariance between stock return and aggregate endowment. The other two components are consumption-share weighted average of the estimation errors of the two investors for each tree’s expected dividend growth rate, respectively.

In equilibrium, the wealth of investor $A$ is $W^A(t) = \frac{1}{\varepsilon^A(t)} E^A_t \left[ \int_t^T e^{-\beta(s-t)} \xi^A(s) c^A(s) ds \right]$. Since $\xi^A(t)$ and $c^A(t)$ are functions of $\varepsilon(t)$ and $\lambda(t)$, we can derive the dynamics of investor $A$’s wealth:

$$
dW^A(t) = E_t \left[ dW^A(t) \right] + \frac{\partial W^A(t)}{\partial \varepsilon(t)} \varepsilon(t) \left[ f(t) \sigma_1 d\omega_1^A(t) + (1 - f(t)) \sigma_2 d\omega_2^A(t) \right]$$

$$- \frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t) \left( g_1(t) d\omega_1^A(t) + g_2(t) d\omega_2^A(t) \right).$$

By equating the diffusion coefficients of Equations (19) and (34), we obtain

$$
\pi^A_{S_{11}}(t) \sigma_{S_{11}}(t) + \pi^A_{S_{21}}(t) \sigma_{S_{21}}(t) = \frac{\partial W^A(t)}{\partial \varepsilon(t)} \varepsilon(t) f(t) \sigma_1 - \frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t) g_1(t),
$$

$$
\pi^A_{S_{12}}(t) \sigma_{S_{12}}(t) + \pi^A_{S_{22}}(t) \sigma_{S_{22}}(t) = \frac{\partial W^A(t)}{\partial \varepsilon(t)} \varepsilon(t) (1 - f(t)) \sigma_2 - \frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t) g_2(t).
$$

We can derive the portfolio holdings of stocks 1 and 2 by investor $A$ by solving Equation (35). Proposition 4 presents the optimal portfolio holdings by investor $A$, and a detailed
proof can be found in the appendix.

**Proposition 4 (Portfolio Holdings)** The optimal portfolio for investor $A$ is

$$\pi_S^A(t) = \begin{bmatrix} \pi_{S1}^A(t) \\ \pi_{S2}^A(t) \end{bmatrix} = \frac{\sigma_{S22}(t)(\frac{\partial W^A(t)}{\partial \epsilon(t)} f(t)\sigma_1 - \frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t)\sigma_1)}{\sigma_{S11}(t)(\frac{\partial W^A(t)}{\partial \epsilon(t)} (1-f(t))\sigma_2 - \frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t)\sigma_2)} \frac{\sigma_{S12}(t)(\frac{\partial W^A(t)}{\partial \lambda(t)} \lambda(t)g_1(t) - \frac{\partial W^A(t)}{\partial \epsilon(t)} f(t)\sigma_1)}{\sigma_{S11}(t)\sigma_{S22}(t) - \sigma_{S12}(t)\sigma_{S21}(t)}$$

(36)

Because the investors in our model have heterogeneous beliefs about expected dividend growth rate, they have different portfolio holdings. Since the optimistic investor expects higher dividend growth than the pessimistic investor, he will hold more stocks than pessimistic investor. In addition, Equation (36) implies that the dividend share $f$ affects portfolio holdings via its impact on the volatility of aggregate endowment and stock volatility.

### 3 Numerical Analysis

In this section, we examine the effects of the disagreements about expected dividend growth of the two trees and the relative dividend share ("size") on the stock expected return and volatility. We use Monte Carlo simulations to perform a numerical analysis. To isolate the disagreement and size effects, we assume that all parameters of the two stocks are the same except their size and degree of investor disagreement.

Table 1 reports the parameter values. Following Dumas, Kurshev, and Uppal (2009), the long-run mean of expected dividend growth $\pi_1 = \pi_1 = 0.015$, and the volatility of dividend $\sigma_1 = \sigma_2 = 0.15$. The coefficient of risk aversion $\gamma$ is 3, the time discount factor $\beta$ is 0.1, and the investment horizon $T = 40$ years. The relative weight of investor $B$ at time $t = 0$, $\frac{\lambda(0)}{1+\lambda(0)}$, is set to be 0.5. That is, the two investors have the same initial weight in the economy.

We assume that the consumption-share average of dividend growth estimated by investors at time $t=0$ equals the actual one, that is, $\frac{1}{1+\lambda(0)^{1/\gamma}}\mu^A_j(0) + \frac{\lambda(0)^{1/\gamma}}{1+\lambda(0)^{1/\gamma}}\mu^B_j(0) = \mu_j(0)$. In this case, the “representative agent” is unbiased about the expected dividend growth, and
thus we can separate the disagreement effect from the aggregate bias effect. To make the disagreement persistent, we set the volatility of expected dividend growth, $\sigma_{\mu j}$, to be 0.02 and the mean-reverting parameter $\kappa_j$ to be 0.1. We choose the value of share of dividend 1, $f$, at time $t = 0$ to be either 0.1 (i.e., stock 1 is small, stock 2 is large) or 0.9 (stock 1 is large, stock 2 is small): when $f = 0.1$, we examine the effect of a small stock’s disagreement on the expected return and return volatility of a large stock; when $f = 0.9$, we examine the effect of a large stock’s disagreement on the expected return and return volatility of a small stock.

The upper panel of Figure 1 plots the return volatility of stock 2 against various degree of investor disagreement $g_1$ about the first stock’s fundamental. The lower panel of Figure 1

Figure 1: Return Volatility and Expected Return of Stock 2 against the Disagreement about the Growth Rate of Dividend 1 Claimed by Stock 1
plots the objective expected return of stock 2 against various degree of investor disagreement $g_1$ about the first stock’s fundamental. In each panel, there are two graphs. The solid graph corresponds to the case of $f = 0.9$ (i.e., stock 1 is large and stock 2 is small), and the dotted graph corresponds to the case of $f = 0.1$. We find that the return volatility of stock 2 increases with $g_1$, investor disagreement for the other stock. The slope of the solid line is higher than that of the dotted line, suggesting that the effect of a large stock’s disagreement on a small stock’s volatility is bigger than the effect of a small stock’s disagreement on a large stock’s volatility. Similarly, the objective expected return of stock 2 increases with investor disagreement $g_1$ about the first stock’s fundamental. This effect is stronger when the first stock is larger. We summarize these patterns in the following hypotheses which will be tested in the next section.

**Hypothesis 1 (Stock Volatility):** There exists a positive relation between the disagreement about the expected growth rate of the dividend claimed by stock 1, $g_1$, and the return volatility of stock 2, and this effect is stronger when stock 1 is larger in size than stock 2.

**Hypothesis 2 (Stock Return):** There exists a positive relation between the disagreement about the expected growth rate of the dividend claimed by stock 1, $g_1$, and the expected return of stock 2, and this effect is stronger when stock 1 is larger in size than stock 2.

## 4 Empirical Analysis

We test the two hypotheses using monthly return data for firms listed on NYSE, AMEX and NASDAQ, obtained from the Center for Research in Security Prices (CRSP). Our proxy for investor disagreement is the dispersion of analyst forecast, obtained from the Thomson Reuters’ IBES Summary History file. Dispersion is measured as the standard deviation of analyst earnings forecasts scaled by the absolute value of the mean earnings forecast. In the theoretical analysis, the size of a firm is measured by its dividend share in the aggregate
endowment. In the empirical analysis, we use the standard proxy for size, the stock’s market capitalization.

Our test methodology is the Fama-MacBeth regressions. In each month’s cross-sectional regression, the dependent variable is next month’s stock return or realized volatility. The independent variables include a stock’s own dispersion of analyst forecast, lagged stock return or volatility. In the model, there are only two stocks, and the disagreement of the large stock has more significant impact on the return and volatility of the small stock. There are thousands of stocks in the data. To test our model’s implication that other stock’s disagreement also matters, we need to, for each stock and in each month, identify another (larger) stock whose dispersion of analyst forecasts will serve as a regressor in the cross-sectional regressions.

In Table 3, we regress a stock’s return or volatility on the disagreement of the corresponding industry leader. For each industry classified by the 2-digit SIC code (HISMG), we label the firm within that industry that has the largest market capitalization as the industry leader. We record its dispersion of analyst forecast, and assign it to all firms belonging to that industry. Table 3 indicates a positive relation between a firm’s stock volatility and its analyst forecast dispersion, and a negative relation between stock return and own firm’s disagreement. Both relations are statistically significant and consistent with previous studies (e.g., Diether, Malloy, and Scherbina (2002)). More importantly, Models 1 and 2 show a significant positive relation between stock volatility and other firm’s disagreement (in this case, the disagreement for the industry leader), consist with our first hypothesis. Models 3 and 4 show a significant positive relation between stock return and disagreement for the industry leader), supporting our second hypothesis.

In Table 4, we perform a further test for Hypothesis 2 using another specification of other firm’s disagreement. In each month, we sort stocks into 50 portfolios based on the book-to-market ratio. For a given stock, we assign to it the dispersion of analyst forecasts for the largest stock belonging to the same $B/M$ sorted portfolio (i.e., $B/M$ leader). Again,
we find a significant positive relation between a stock’s return next month and the current disagreement for the $B/M$ leader. At the same time, there is a negative relation between stock return and own firm’s disagreement.

5 Conclusion

We extend CLS to an economy where two groups of investors have different beliefs about the expected dividend growth of two stocks. The state price density in the model is affected by investor disagreements for both stocks, especially that for the larger stock. The model predicts a positive relation between the expected return or return volatility of one stock and investor disagreement about the other stock, even when the two fundamentals are uncorrelated. This spillover effect is stronger when one examines the impact of disagreement about a large stock on the return or volatility of a small stock. These novel model implications are supported by our empirical analysis.
References


Appendix: Proof of Proposition 3

The price of stock 2 can be expressed as

\[ S_2(t) = \frac{e(t)^\gamma}{(1+\lambda(t))^{1/\gamma}} E_t^A \left[ \int_t^T e^{-\beta(s-t)} \left( \frac{1+\lambda(s)^{1/\gamma}}{\sigma(s)} \right)^\gamma D_2(s) \, ds \right] . \]

We have

\[ d\mu_j^A(t) = \kappa_j \left( \pi_j - \mu_j^A(t) \right) \, dt + \frac{\overline{v}_j}{\sigma_j} d\omega_j^A(t) . \]

Solving the above stochastic differential equation yields

\[ \mu_j^A(s) = e^{\kappa_j(t-s)} \mu_j^A(t) + \left( 1 - e^{\kappa_j(t-s)} \right) \pi_j + \int_t^s e^{\kappa_j(u-s)} \overline{v}_j \frac{1}{\sigma_j} d\omega_j^A(u) . \]

Taking Mallavine derivative of dividend process \( \mu_j^A \), we have \( D_{t,j} \mu_j^A(s) = \frac{e^{\kappa_j(t-s)} \overline{v}_j}{\sigma_j} \). The dividend process \( D_j \) satisfies

\[ D_j(t) = D_j(0) \exp \left\{ \int_0^t \left( \mu_j^A(u) - \frac{1}{2} \sigma_j^2 \right) \, du + \int_0^t \sigma_j d\omega_j^A(u) \right\} . \]

Taking Mallavine derivative of dividend process \( D_2 \), we have \( D_{t,1}^A D_2(s) = 0 \), and

\[ D_{t,2}^A D_2(s) = D_2(s) \left( \sigma_2 + \int_s^t D_{t,2}^A \mu_2^A(u) \, du \right) = D_2(s) \left( \sigma_2 + \int_s^t \frac{e^{\kappa_2(t-u)} \overline{v}_2}{\kappa_2 \sigma_2} du \right) . \]

Similarly, we have \( D_{t,2}^A D_1(s) = 0 \), and \( D_{t,1}^A D_1(s) = D_1(s) \left( \sigma_1 + \frac{1 - e^{\kappa_1(t-u)} \overline{v}_1}{\kappa_1 \sigma_1} \right) . \)

Solving stochastic differential equations of \( \lambda \), we have

\[ \lambda(t) = \lambda(0) \exp \left\{ - \int_0^t \frac{1}{2} \left( g_1(u)^2 + g_2(u)^2 \right) \, du - \int_0^t g_1(u) \, d\omega_1^A(u) - \int_0^t g_2(u) \, d\omega_2^A(u) \right\} . \]

Taking Mallavine derivative of dividend process \( \lambda \), we have

\[ D_{t,j}^A \lambda(s)^{1/\gamma} = \frac{1}{\gamma} \lambda(s)^{1/\gamma-1} D_{t,j}^A \lambda(s) = - \frac{1}{\gamma} \lambda(s)^{1/\gamma} g_j(t) . \]

Similarly, the Mallavine derivative of aggregate endowment, \( \varepsilon \), is

\[ D_{t,j}^A \varepsilon(s) = D_{t,j}^A D(s) = D_j(s) \left( \sigma_j + \frac{1 - e^{\kappa_j(t-u)} \overline{v}_j}{\kappa_j \sigma_j} \right) . \]
Therefore, we have

\[
D^A_{t,j} D^A_t \xi^A(s) = D^A_{t,j} \left( \frac{1 + \lambda(s)^{1/\gamma}}{\sigma(s)} \right)^\gamma = \gamma \left( \frac{1 + \lambda(s)^{1/\gamma}}{\sigma(s)} \right)^{\gamma - 1} \left( \frac{D^A_{t,j} \lambda(s)^{1/\gamma}}{\sigma(s)} + \left( 1 + \lambda(s)^{1/\gamma} \right) D^A_{t,j} \frac{1}{\sigma(s)} \right)
\]

\[
= \gamma \xi^A(s) \left( \frac{D^A_{t,j} \lambda(s)^{1/\gamma}}{1 + \lambda(s)^{1/\gamma}} - \frac{D^A_{t,j} \sigma(s)}{\sigma(s)} \right),
\]

and then

\[
D^A_{t,1} \xi^A(s) = -\xi^A(s) \left[ \psi(s) g_1(t) + \gamma f(s) \left( \sigma_1 + \frac{(1 - e^{\kappa_1 (t - s)}) \psi_1}{\kappa_1 \sigma_1} \right) \right],
\]

\[
D^A_{t,2} \xi^A(s) = -\xi^A(s) \left[ \psi(s) g_2(t) + \gamma (1 - f(s)) \left( \sigma_2 + \frac{(1 - e^{\kappa_2 (t - s)}) \psi_2}{\kappa_2 \sigma_2} \right) \right].
\]

where \( \psi(t) = \frac{\lambda(t)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \). Using the Clark-Ocone formula, we can derive the stock volatility (e.g., the two diffusion items) of second stock given by:

\[
\sigma_{S2,j} (t) = \theta^j (t) + \frac{E^A_t \int_t^T D^A_{t,j} (\xi^A(s) D^A_{2}(s)) ds}{E^A_t \int_t^T \xi^A(s) D^A_{2}(s) ds} = \theta^j (t) + \frac{E^A_t \int_t^T [D^A_{2}(s) D^A_{t,j} \xi^A(s) + \xi^A(s) D^A_{t,j} D^A_{2}(s)] ds}{E^A_t \int_t^T \xi^A(s) D^A_{2}(s) ds}.
\]

That is,

\[
\sigma_{S2,1} (t) = \theta^1 (t) - \frac{E^A_t \int_t^T e^{-\beta(s-t)} \xi^A(s) D^A_{2}(s) \left[ \psi(s) g_1(t) + \gamma f(s) \left( \sigma_1 + \frac{(1 - e^{\kappa_1 (t - s)}) \psi_1}{\kappa_1 \sigma_1} \right) \right] ds}{E^A_t \int_t^T e^{-\beta(s-t)} \xi^A(s) D^A_{2}(s) ds} ,
\]

\[
\sigma_{S2,2} (t) = \sigma_2 + \theta^2 (t) - \frac{E^A_t \int_t^T e^{-\beta(s-t)} \xi^A(s) D^A_{2}(s) \left[ \psi(s) g_2(t) + \gamma (1 - f(s)) \sigma_2 + (\gamma (1 - f(s)) - 1) \right] \left( \frac{1 - e^{\kappa_2 (t - s)}) \psi_2}{\kappa_2 \sigma_2} \right) ds}{E^A_t \int_t^T e^{-\beta(s-t)} \xi^A(s) D^A_{2}(s) ds} .
\]

\[
\mu_{S2} (t) = r(t) + \sigma_{S21} (t) \left[ \gamma f(t) \sigma_1 + \psi(t) g_1(t) - \frac{\mu^1_{S2}(t) - \mu_1(t)}{\sigma_1} \right] + \sigma_{S22} (t) \left[ \gamma (1 - f(t)) \sigma_2 + \psi(t) g_2(t) - \frac{\mu^2_{S2}(t) - \mu_2(t)}{\sigma_2} \right]
\]

\[
= r(t) + \sigma_{S21} (t) \left[ \gamma f(t) \sigma_1 \sigma_{S21} (t) + (1 - f(t)) \sigma_2 \sigma_{S22} (t) \right] + \frac{\sigma_{S21}(t)}{\sigma_1} \left[ \mu_1(t) - [(1 - \psi(t)) \mu^A_1(t) + \psi(t) \mu^B_1(t)] \right] + \frac{\sigma_{S22}(t)}{\sigma_2} \left[ \mu_2(t) - [(1 - \psi(t)) \mu^A_2(t) + \psi(t) \mu^B_2(t)] \right].
\]

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Proof of Proposition 4

Following the methodology of Buraschi and Jiltsov (2006), we calculate the joint distribution of aggregate endowment and relative weight of investor B.

(1) Marginal distribution of relative weight of investor B, $\lambda(t)$: the stochastic relative weight $\lambda(t)$ is given by

$$
\lambda(s) = \lambda(t) \exp \left\{ - \int_{t}^{s} \left( g_1(u)^2 + g_2(u)^2 \right) du \right\} \times \exp \left\{ - \int_{t}^{s} g_1(u) d\omega_1^A(u) - \int_{t}^{s} g_2(u) d\omega_2^A(u) \right\}
$$

$$
\lambda = \lambda(t) \exp \left( M_\lambda(s, t) - \sqrt{V_\lambda(s, t)Z_\lambda} \right), \quad Z_\lambda \sim N(0, 1).
$$

(2) Marginal distribution of aggregate endowment, $\varepsilon(t)$: the aggregate endowment $\varepsilon(t)$ is given by

$$
\varepsilon(s) = \varepsilon(t) \exp \left\{ \int_{t}^{s} \left[ f(u) \mu_1^A(u) + (1 - f(u)) \mu_2^A(u) - \frac{1}{2} \left( f(u) \sigma_1^2 - \frac{1}{2} (1 - f(u)) \sigma_2^2 \right) \right] du \right\}
$$

$$
\times \exp \left\{ \int_{t}^{s} f(u) \sigma_1 d\omega_1^A(u) + \int_{t}^{s} (1 - f(u)) \sigma_2 d\omega_2^A(u) \right\}
$$

$$
= \varepsilon(t) \exp \left( M_\varepsilon(s, t) - \sqrt{V_\varepsilon(s, t)Z_\varepsilon} \right), \quad Z_\varepsilon \sim N(0, 1).
$$

(3) Joint distribution of $\lambda(t)$ and $\varepsilon(t)$:

$$
\begin{pmatrix}
\log \lambda(s) \\
\log \varepsilon(s)
\end{pmatrix}
\sim N
\begin{pmatrix}
M_\lambda(s, t) & V_\lambda(s, t) & \text{Cov}_{\lambda, \varepsilon}(s, t) \\
M_\varepsilon(s, t) & V_\varepsilon(s, t)
\end{pmatrix}.
$$

(4) The wealth of investor A is

$$
W^A(t) = \frac{1}{\xi^A(t)} E_t^A \left[ \int_{t}^{T} e^{-\beta(s-t)}\xi^A(s) e^A(s) ds \right] = E_t^A \left[ \int_{t}^{T} e^{-\beta(s-t)} e^{M_\lambda(s, t)-\sqrt{V_\lambda(s, t)Z_\lambda}} ds \right]
$$

$$
= \frac{E_t^A \left[ \int_{t}^{T} e^{-\beta(s-t)} F_\lambda(s, t)^{1-\gamma} F_\varepsilon(s, t)^{\gamma - 1} ds \right]}{\varepsilon(t)^{-\gamma}(1+\lambda(t)^{1/\gamma})^\gamma}.
$$

where

$$
F_\lambda(t, s) = F_\lambda(t, s, \lambda(t), Z_\lambda) = 1 + \left[ \lambda(t) e^{M_\lambda(s, t)-\sqrt{V_\lambda(s, t)Z_\lambda}} \right]^{1/\gamma} = 1 + \left[ \lambda(t) e^{H_\lambda(s, t)} \right]^{1/\gamma},
$$

$$
F_\varepsilon(t, s) = F_\varepsilon(t, s, \varepsilon(t), Z_\varepsilon) = \varepsilon(t) e^{H_\varepsilon(s, t)},
$$

where $H_\lambda(s, t) = M_\lambda(s, t) - \sqrt{V_\lambda(s, t)Z_\lambda}$ and $H_\varepsilon(s, t) = M_\varepsilon(s, t) - \sqrt{V_\varepsilon(s, t)Z_\varepsilon}$.
The numerator of \( W^A (t) \) can be written as
\[
E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \, ds \right] = \int_t^T \int_{-\infty}^\infty F_\varepsilon (t, s, \varepsilon (t), Z_\varepsilon) \gamma^{-1} F_\lambda (t, s, \lambda (t), Z_\lambda) \gamma^{-1} N (Z_\varepsilon, Z_\lambda, \rho (t, s)) \, dZ_\varepsilon dZ_\lambda ds,
\]
where \( \rho (t, s) = \frac{\text{Cov}_{\varepsilon, \lambda} (s, t)}{\sqrt{\text{Var}_\varepsilon (s, t)} \sqrt{\text{Var}_\lambda (s, t)}} \).

Therefore, the derivative of \( W^A (t) \) with respect to \( \lambda (t) \) is
\[
\frac{\partial W^A (t)}{\partial \lambda_t} = \frac{\varepsilon (t) \gamma}{(1+\lambda (t)^{1/\gamma})^2} \left[ \frac{\partial}{\partial \lambda_t} E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \, ds \right] \right]
\]

where
\[
\frac{\partial}{\partial \lambda_t} E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \, ds \right]
= E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} \frac{\partial}{\partial \lambda_t} \left( F_\lambda (t, s) \gamma^{-1} \right) \, ds \right]
= \frac{\gamma-1}{\gamma} E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \left[ \lambda (t) e^{H_\lambda (s,t)} \right] \gamma^{-2} \lambda (t)^{1/\gamma-1} \, ds \right],
\]
and, similarly, the derivative of \( W^A (t) \) with respect to \( \varepsilon (t) \) is
\[
\frac{\partial W^A (t)}{\partial \varepsilon_t} = \frac{\varepsilon (t) \gamma}{(1+\lambda (t)^{1/\gamma})^2} \left[ \frac{\partial}{\partial \varepsilon_t} E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \, ds \right] \right]
\]

where
\[
\frac{\partial}{\partial \varepsilon_t} E^A_t \left[ \int_t^T e^{-\beta(s-t)} F_\varepsilon (t, s) \gamma^{-1} F_\lambda (t, s) \gamma^{-1} \, ds \right] = E^A_t \left[ \int_t^T e^{-\beta(s-t)} \varepsilon (t) \gamma e^{(1-\gamma) H_\varepsilon (s,t)} F_\lambda (t, s) \gamma^{-1} \, ds \right].
\]

The optimal holding of stock 2 for investor \( A \) is
\[
\pi^A_{S2} (t) = \frac{\frac{\partial W^A (t)}{\partial \varepsilon (t)} \cdot \varepsilon (1-f (t)) \sigma_{S11} (t) \sigma_{S21} (t) \sigma_{S12} (t) \sigma_{S22} (t) \sigma_{S1} + \frac{\partial W^A (t)}{\partial \lambda_t} \lambda (t) \sigma_{S12} (t) g_1 (t) - \sigma_{S11} (t) g_2 (t)}{\sigma_{S11} (t) \sigma_{S22} (t) - \sigma_{S12} (t) \sigma_{S21} (t)}.
\]

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## Table 1: Parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>Symbols</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters for aggregate endowment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dividend share of asset 1 at t=0</td>
<td>$f(0)$</td>
<td>0.1, 0.9</td>
</tr>
<tr>
<td>Long-run mean of growth of dividend $j$</td>
<td>$\pi_j$</td>
<td>0.015</td>
</tr>
<tr>
<td>Standard deviation of dividend $j$</td>
<td>$\sigma_j$</td>
<td>0.13</td>
</tr>
<tr>
<td>Standard deviation of growth of dividend $j$</td>
<td>$\sigma_{\mu j}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Mean-reverting parameter of dividend $j$</td>
<td>$k_j$</td>
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</tr>
<tr>
<td>Investment horizon (years)</td>
<td>$T$</td>
<td>40</td>
</tr>
<tr>
<td><strong>Parameters for investors</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of Risk Aversion</td>
<td>$\gamma$</td>
<td>3</td>
</tr>
<tr>
<td>Time discount factor</td>
<td>$\beta$</td>
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<tr>
<td>Weight of investor B at t=0</td>
<td>$\frac{\lambda(0)}{1+\lambda(0)}$</td>
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<td>Disagreement about stock 2</td>
<td>$g_2$</td>
<td>0.25</td>
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Table 2: Variable Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description (j = 1, 2)</th>
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</thead>
<tbody>
<tr>
<td>$D_j$</td>
<td>Dividend $j$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Aggregate endowment</td>
</tr>
<tr>
<td>$f$</td>
<td>Share of $D_1$</td>
</tr>
<tr>
<td>$\mu_j$</td>
<td>Expected growth rate of $D_j$</td>
</tr>
<tr>
<td>$\mu_j^i$</td>
<td>Investor $i$’s posterior belief about $\mu_j$</td>
</tr>
<tr>
<td>$k_j$</td>
<td>Mean-reverting parameter of $\mu_j$</td>
</tr>
<tr>
<td>$\pi_j$</td>
<td>Long-run mean of $\mu_j$</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>Volatility of $D_j$</td>
</tr>
<tr>
<td>$\sigma_{\mu_j}$</td>
<td>Volatility of $\mu_j$</td>
</tr>
<tr>
<td>$\omega_j$</td>
<td>Fundamental risk (of $D_j$)</td>
</tr>
<tr>
<td>$\omega_{\mu_j}$</td>
<td>Brownian motion (of $\mu_j$)</td>
</tr>
<tr>
<td>$\omega_j^i$</td>
<td>Investor $i$’s innovation (of $D_j$)</td>
</tr>
<tr>
<td>$\nu_j^i$</td>
<td>Conditional variance of investor $i$’s belief about $\mu_j$</td>
</tr>
<tr>
<td>$\nu_j$</td>
<td>Stationary variance of investor $i$’s belief about $\mu_j$</td>
</tr>
<tr>
<td>$g_j$</td>
<td>Disagreement between two investors about $\mu_j$</td>
</tr>
<tr>
<td>$\phi_j$</td>
<td>Mean-reverting parameter of $g_j$</td>
</tr>
<tr>
<td>$B$</td>
<td>Price of riskless asset</td>
</tr>
<tr>
<td>$S_j$</td>
<td>Price of stock $j$</td>
</tr>
<tr>
<td>$\mu_{S_j}$</td>
<td>Objective expected return of stock $j$</td>
</tr>
<tr>
<td>$\sigma_{S_jk}$</td>
<td>Return volatility of stock $j$ caused by fundamental risk $k$</td>
</tr>
<tr>
<td>$\mu_{S_j}^i$</td>
<td>Investor $i$’s subjective expected return of stock $j$</td>
</tr>
<tr>
<td>$\xi^i$</td>
<td>Investor $i$’s state-price density</td>
</tr>
<tr>
<td>$r$</td>
<td>Riskfree rate</td>
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<tr>
<td>$\theta_j^i$</td>
<td>Market price of fundamental risk $j$ perceived by investor $i$</td>
</tr>
<tr>
<td>$c^i$</td>
<td>Investor $i$’s consumption</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of risk aversion</td>
</tr>
<tr>
<td>$\pi_{S_j}^i$</td>
<td>Investor $i$’s portfolio of stock $j$</td>
</tr>
<tr>
<td>$W^i$</td>
<td>Investor $i$’s wealth</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Consumption share of investor $B$</td>
</tr>
<tr>
<td>$\psi^i$</td>
<td>Lagrangian multiplier for investor $i$’s static budget constraint</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Relative weight of investor $B$</td>
</tr>
</tbody>
</table>
This table reports the results of monthly Fama-MacBeth regressions of the firm’s volatility and return of the next period on this period-ending firm’s own dispersion $DISP$ and the dispersion of the industry leader respectively. The dependent variables are the next period volatility (Model 1 and Model 2) and return (Model 3 and Model 4) respectively. $Firm\ DISP$ is measured as the standard deviation of earnings forecasts scaled by the absolute value of the mean earnings forecast obtained from IBES. $Industry\ Leader\ DISP$ is the dispersion of the largest firm in the same industry based on the 2-digit SIC code. Volatility is the historical volatility of the last month. Return is the equity return of the last month. The sample period is from January 1976 to December 2011. The numbers in the brackets are $t$-statistics. * (resp. ** and *** ) denotes significance at 10% (resp. 5% and 1%) level.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility at $t+1$</td>
<td>486.74***</td>
<td>201.77***</td>
<td>−10.50**</td>
<td>−12.01**</td>
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<tr>
<td></td>
<td>(43.49)</td>
<td>(23.66)</td>
<td>(−2.02)</td>
<td>(−2.48)</td>
</tr>
<tr>
<td>Industry $DISP$</td>
<td>29.57**</td>
<td>19.68**</td>
<td>9.71***</td>
<td>9.70***</td>
</tr>
<tr>
<td></td>
<td>(2.52)</td>
<td>(2.37)</td>
<td>(2.94)</td>
<td>(2.96)</td>
</tr>
<tr>
<td>Volatility at $t$</td>
<td>0.57***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(89.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return at $t$</td>
<td></td>
<td>−0.03***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−5.42)</td>
<td></td>
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</tr>
<tr>
<td>Constant</td>
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<td>0.16***</td>
<td>0.01***</td>
<td>0.01***</td>
</tr>
<tr>
<td></td>
<td>(64.58)</td>
<td>(44.93)</td>
<td>(5.24)</td>
<td>(5.42)</td>
</tr>
</tbody>
</table>
Table 4: The $B/M$ Portfolio Leader’s Dispersion and the Firms’ Return

This table reports the results of monthly Fama-MacBeth regressions of next month’s stock return on current dispersion of analyst forecasts for the own firm and as well as that for the $B/M$ Leader. $Firm\ DISP$ is measured as the standard deviation of earnings forecasts scaled by the absolute value of the mean earnings forecast. In each month, we sort stocks into 50 portfolios based on the book-to-market ratio. For a given stock, $B/M\ Leader\ DISP$ is the dispersion of the largest stock belonging to the same $B/M$ sorted portfolio. The sample period is from January 1980 to September 2013. The numbers in the brackets are $t$-statistics. * (resp. ** and *** ) denotes significance at 10% (resp. 5% and 1%) level.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Return at $t + 1$</td>
<td>Return at $t + 1$</td>
<td>Return at $t + 1$</td>
<td>Return at $t + 1$</td>
</tr>
<tr>
<td>$Firm\ DISP$</td>
<td>$-0.16^{***}$</td>
<td>$-0.16^{***}$</td>
<td>$-0.20^{***}$</td>
<td>$-0.19^{***}$</td>
</tr>
<tr>
<td></td>
<td>($-2.92$)</td>
<td>($-2.90$)</td>
<td>($-3.58$)</td>
<td>($-3.55$)</td>
</tr>
<tr>
<td>$B/M\ Leader\ DISP$</td>
<td>$2.00^{***}$</td>
<td>$1.78^{***}$</td>
<td>$0.79^{***}$</td>
<td>$0.79^{***}$</td>
</tr>
<tr>
<td></td>
<td>($6.60$)</td>
<td>($6.00$)</td>
<td>($3.09$)</td>
<td>($3.10$)</td>
</tr>
<tr>
<td>Return at $t$</td>
<td>$-0.04^{***}$</td>
<td></td>
<td>$-0.04^{***}$</td>
<td></td>
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<tr>
<td></td>
<td>($-7.17$)</td>
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<td>($-7.59$)</td>
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<tr>
<td>$B/M$</td>
<td>$1.11^{***}$</td>
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<td>$0.99^{***}$</td>
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<td>($6.89$)</td>
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<tr>
<td>$Size$</td>
<td>$0.03$</td>
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<td></td>
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<td>($0.89$)</td>
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</tr>
<tr>
<td>Constant</td>
<td>$1.19^{***}$</td>
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</tr>
<tr>
<td></td>
<td>($4.70$)</td>
<td>($4.98$)</td>
<td>($0.04$)</td>
<td>($0.08$)</td>
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</tbody>
</table>